Cognitive System Formalism: Analysis of an Architecture for Human/Robot Interactions

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Abstract

We have designed a Cognitive Systems Formalism (CSF) in order to simplify, analyze, compare and predict in a formal way the behaviors of autonomous and embodied *Cognitive Systems* (CS) involved in a dynamical loop with their environment. This formalism can apply when the environment contains other cognitive systems. The present paper aims to explain in details how the CSF can be used for the theoretical study of an architecture which has been designed, implemented and tested by another lab. All along the paper we will attempt to show two things. The first one is how the CSF enables to study this specific architecture dedicated to human/robot interactions; to retrieve theoretically the experimental results of its designers Hosada et al. and Nagai et al.; and to enrich the original architecture so as to increase its interaction capabilities suggesting few changes focused on the learning dynamics. Second, we will show that this formal study of a specific architecture enlights general principles concerning human/robots interactions (for instance characterizing the time scale of the interaction loop) and thus enrich the CSF's framework and rules and may enable more efficient and relevant future studies of others architectures. This paper goes one step further toward an ideal tool for CS studies.

1 Introduction

Nowadays, there is an increasing variety of functional robotic systems. Most of these systems are claimed to be original, involving higher capabilities and performances than the others. If we want to capitalize on all these results, we must be able to determine in a given architecture what is innovative and really different from the others. We must also be able to detect when two architectures or subpart of an architecture are equivalent even if they appear as quite different because of the way they are written. Hence, some formal tools (as simple as possible) should be introduced to enlight the principles which make different architectures working, independently from their original writing and technical implementation. Cognitive sciences have the same problem: experimental results in neurobiology and psychology lead to conceptual models which are often quite far from computational models and architectures. There is the same requirement for a formal tool, which could enable to explain in few equations an idea that need usually several pages of textual development to be justified. We believe that one or several mathematical tools could be developed to study both robotic and natural systems. These formalisms should translate the intuitive expertise used by the roboticians, the psychologists and the neurobiologists. In order to make one step toward the design of such an ideal tool, we have proposed the Cognitive System Formalism (CSF), including a theoretical framework and simplification rules. The CSF may enable to simplify, analyze, compare and predict in a formal way the behaviors of autonomous and embodied *Cognitive Systems*[1] (CS) embedded in their environment, especially if the environment contains other cognitive systems [Gaussier, 2001, Gaussier et al., 2004]. The CSF is based on the deep structural link that exists between Sensorial data[2] and Action in any autonomous and embodied system. Previous studies on robotic or human development have shown that many cognitive capabilities result from the agent / environment interaction (as a single dynamical system) and the emergent relations between sensation and action [Nadel et al., 2005, Maillard et al., 2005, Pfeifer and Scheier, 1999, Berthoz, 1997, Gaussier and Zrehen, 1995]. Learning capabilities in autonomous and embodied cognitive systems induce an evolution and an adaptation of their internal dynamics. This capacity of "changing" along time is the key problem for the study of CS since the changes must be taken into account in the study of the dynamics. The CSF allows to formalize easily the interaction between the studied agent and its environment or another agent. Moreover, the CSF points out the evolution of the interaction before and after learning stabilization. It is one of the major differences between our formalism and classical AI studies in which systems are only input/output processes (no dynamical loops). Hence, the study of the agent behavior implies the study of the dynamic of the "agent-environment" system. In that way, the CSF is a theoretical approach to bridge the gap between dynamical systems theories, long life learning and experiments in robotics and developmental psychology.

In previous papers concerning the CSF, we have defined the first principles and rules of the formalism. [Gaussier, 2001] defined elementary operations (addition, composition...) and used them to

[1] "*Cognitive systems*" should be understood here in the sense of [Mataruna and Varela, 1980] and more recently [Bourgine and Stewart, 2004]: Let *type A* interactions be interactions that have consequences for the internal state of the system and let *type B* interactions be interactions that have consequences for the state of the (proximal) environment or the relation of the system to its environment. Then, *a system is cognitive if and only if type A interactions serve to trigger type B interactions in a specific way, so as to satisfy a viability constraint.*

[2] Previously, in the *PerAc* architecture (Perception Action)[Gaussier and Zrehen, 1995] we did not separate *Perception* from the simple *Sensory* flow of information. We prefer now to devote the use of *Perception* to the description of the global *Sensory-Motor* dynamics [Maillard et al., 2005]

simplify a set of equations describing a planning CS. The CSF was aimed to be a tool to compare the complexity of different control architectures in term of an energy measure. This aspect of the formalism is not addressed in the present paper but is still one of the way to go farther with the formalism. In [Gaussier et al., 2003, Gaussier et al., 2004], two types of simplification rules are defined and used: one independent from the relation between sensation and action and the other constrained by this relation and thus constrained by the convergence of learning (see section 2, *Basic principles of the formalism*). These two types of simplifications are key elements of the CSF framework: First they give us the constraints enabling learning convergence. Second they allow to predict some aspects of the after learning behavior of the agent (if the constraints were satisfied during learning). We have applied the CSF to build and study a simple theoretical model of the development of the capability to express and recognize more and more complex facial expressions. First, we proposed a simple architecture. Second, we simplified our theoretical architecture and analyzed some of its properties. Third, we shown that this architecture was able to learn the bidirectional association between an internal "emotion" and its associated facial expression. To demonstrate this feature, we have proved that learning was only possible if another agent acts as a mirror. Thus, we have retrieve theoretically human development results which support the idea that the mirroring acts according to a process similar to that of a social biofeedback (neonates self-imitation [Rochat, 2002] and imitation of another action [Nadel et al., 2004]): mirroring enables to learn something about him/herself via the social environment.

We believe that the CSF approach can be directly applied to other problems of the same level of complexity. One of the goals of the present paper is to test the CSF on an architecture completely new for us and conceived to allow Joint Attention Learning (JAL), a problem we are not specialists. Even if the original writing and embodiment of the JAL architecture has not much in common with the architecture studied in our lab, the CSF should enable us to analyze it and to bring commentaries and enrichment. Many teams have conceived JAL architectures and proposed different models of joint attention (JA) [Breazeal and Scassellati, 1999, Kozima et al., 2003, Nagai et al., 2003, Lau and Triesch, 2004, Morita et al., 2004, Nagai, 2005a, Sumioka et al., 2005]: our bet is that the analyses allowed by our formalism are general enough to account for the various models of JA (see figure 1, an example of an experimental setup for JAL architecture). Among all these JAL architectures, we have chosen to formally analyze a particular one, already implemented and tested [Nagai et al., 2003, Hosoda et al., 2004]. In [Nagai et al., 2003], joint attention is defined by *looking at the same object that someone else is looking at*, and in [Hosoda et al., 2004] it is defined by *a process to attend to the object that the other attends*. To fit these definitions and their associated JAL architectures, we will assume joint attention is *the capacity to watch the same object than someone else by controlling the other gaze direction*. Thus, JAL architecture is supposed, as a cognitive system, to improve its capability to exhibit a dynamics consisting, at the end of the learning process, in gazing at the same object than the demonstrator. To sum up, the robot, must learn associations between the objects positions and the human gaze direction just by looking alternatively at the objects on the table and at the human caregiver. After this learning, the robot must be able to look at the human's face and then to look at the same object as the human is gazing at. In the experimental setup we will study, the robot is sitting in front of a table where 3 objects are present. A human caregiver faces the robot. According to our CSF theoretical approach, the problem here is not to know *how this dynamics could work* but rather *how an architecture may evolve through time from a dynamics that does not depend on JA at all toward a dynamics whose attraction basin is JA*.

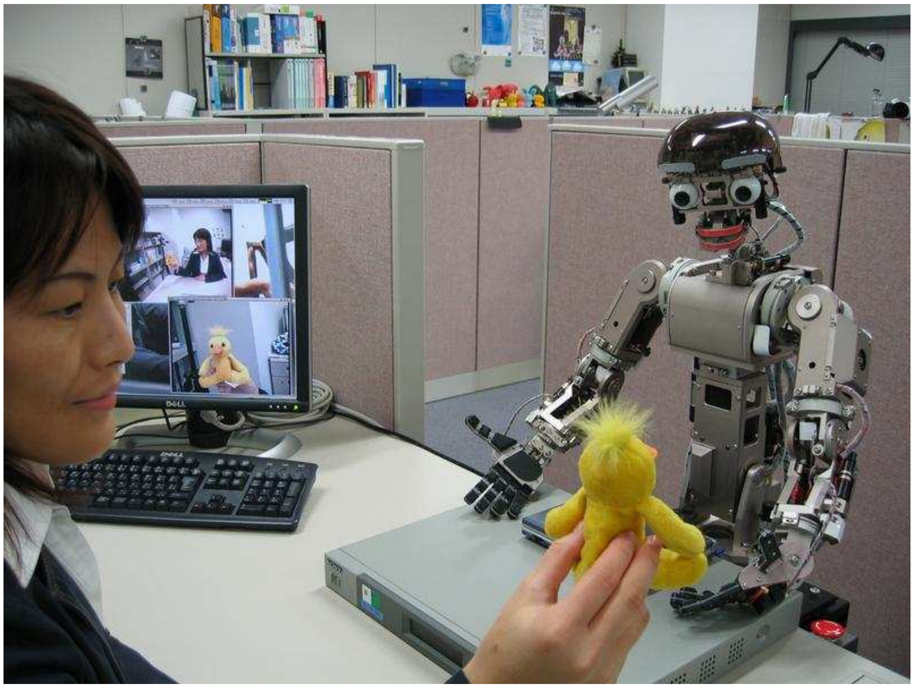

Figure 1: Yukie Nagai interacting with Infanoid. Joint Attention Learning based on Self-Other Motion Equivalence [Nagai, 2005b].

The present paper aims to explain in details how the CSF can be used for the theoretical study of an architecture which has been designed, implemented and tested by another lab [Hosoda et al., 2004, Nagai et al., 2003]. All along the paper, we will attempt to show two things. The first one is how the CSF enables to study this specific architecture dedicated to human/robot interactions; to retrieve theoretically the experimental results of Hosada et al. and Nagai et al.; and to enrich the original architecture so as to increase its interaction capabilities. Second, we will show that this formal study of a specific architecture enlights general principles concerning human/robots interactions (for instance characterizing the time scale of the interaction loop) and thus enrich the CSF's framework and rules and may enable more efficient and relevant future studies of others architectures. In section 2 (*Basic principles and rules of the formalism*) we will introduce the CSF, detailing the different steps necessary to perform the formal analysis of an architecture: formal writing of the architecture, Constraints Enabling Learning (CEL), simplification rules, additional *constraints on the dynamics*, and learning capabilities. In section 3, the schematic architecture and the associated algorithms proposed in [Nagai et al., 2003, Hosoda et al., 2004] will be translated in our mathematical formalism. The equations obtained will represent both the flows of information, the decision making and the learning mechanisms used in the architecture fig 14. In section 4, we will analyze theoretically the JAL architecture. The architecture will be immersed in its environment. The equations will represent the whole interaction loop and will be simplified using simplifications independent from any Sensory-Motor coupling. In section 5, we will determinate the constraints enabling the learning convergence. In section 6 these constraints associated to stable states of Sensory-Motor coupling will first be used to specify the dynamics of interaction between the JAL architecture and its environment (the objects and the caregiver). Next, this dynamics specification will lead to propose changes in the JAL architecture such as "learning through a temporal window" which may enlarge the dynamics of interaction leading to the convergence of learning. We will then conclude this study and stress more general issues balancing the results of this paper, our previous results and the direction of the ambitious goal we attempt to follow.

2 Basic principles and rules of the formalism

A Cognitive System (CS, see foot-note 1), for instance a JAL architecture, is supposed to be made of several interconnected elements: some associated with input information (via specific "sensor" elements), some with output information (command of actions – via "actuators") and some dedicated to intermediate processes. A CS interacts with its environment in a closed interaction loop. See, for instance figure 2 which shows a prototypical control architecture for a cognitive system.

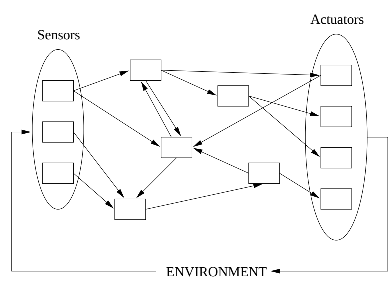

Figure 2: Prototypical architecture that can be manipulated by our formalism.

Our formalism aims both to represent and analyze such a system. Each part of an architecture should be represented as a specific "Sensory-Action" coupling which may be characterized by a wave function evolving through time. This issue is not addressed in the present paper but it is one way which should be investigated [Gaussier, 2001, Maillard et al., 2005]. For that purpose, our mathematical formalism has taken inspiration from quantum mechanics. Practically, we have chosen to represent the input and output of a CS by vectors in the "bra-ket" notation. It is considered that any processing element of a CS takes an input vector $|x\rangle$ (column vector of size m with $|x\rangle \in R_+^m$) and filters it according to a matrix of weights W representing the functional links with the input group. The multiplication of the vector $|x\rangle$ by a matrix W is written $|y\rangle = W|x\rangle$ with $|y\rangle \in R_+^n$ for a matrix of size $n \times m$. Two main types of W matrices are distinguished according to their learning capabilities: *Unconditional* matrix U for reflex mechanisms (no learning) and *Adaptive* matrix A which are used for pattern matching processing, categorization... or all the other ways of filtering performed by learning. Moreover, to take into account the fact that a processing element may be modified according to a non-linear function and a pattern of interactions between the elements of the same block, a non-linear operator k is introduced to characterize this processing. It can be either a simple processing, such as a scalar product or a distance measure, or a more complex operator such as an "If... then... else..." statement (hard decision making), a pattern of lateral interactions in the case of a competitive structure or a recurrent feedback (at short or long range). It is important to notice that k defines both the way to use the weight matrices in order to compute the output and the way to modify the matrices according to a given learning criteria. Figure 3 shows the schemas of two processing elements, a reflex and an adaptive mechanism, and their associated writing with the formalism.

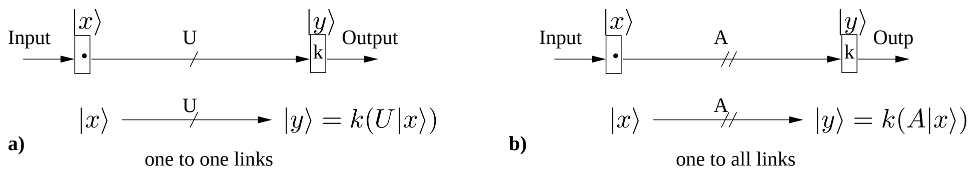

Figure 3: Writing of two basic processing units. The upper part of the figure is the graphical representation and the lower part of the figure is the formal notation. Arrows with one stroke represent "one to one" reflex connections and arrows with two parallel strokes represent "one to all" modifiable connections. a) *Unconditional* "one to one" connections (used as a reflex link) between two groups. b) *Adaptive* "one to all" connections (used as a categorization mechanism) between two groups. $|x\rangle$ and $|y\rangle$ are input and output vectors. U and A are the matrices of weights. k is the operator which controls both the way to combine the weight matrices with the input vector so as to obtain the output vector and the learning process. In fig.a), k is only a competition mechanism where as in fig.b), some auto-organization mechanism must be taken into account for the learning process (a "kohonen map" for instance).

One of the purpose of our formalism is to transform a CS architecture in an equivalent architecture simpler to analyze and to understand. Thus we define, in a first approximation, the equivalence between two architectures (*Architecture1*≡*Architecture2*) as follows: two architectures are equivalent if they have the same "behavioral attractors" [Beer, 1995, Gaussier et al., 2003, Revel and Nadel, 2004] for a given environment. In that perspective, a control architecture cannot be studied without taking into account the nature of the interactions with the environment and the dynamics which will enable the convergence of learning. Let $|x_1\rangle$ and $|x_2\rangle$ be two input vectors and $|y\rangle$ be an output vector. Let U and A be respectively unconditional and adaptive links. Let k be an operator which on one hand computes the vector $|y\rangle$ selecting the winning component of the vector $U|x_1\rangle + A|x_2\rangle$ and on the other hand processes the conditional learning of the link A depending on the output $|y\rangle$. Thus, the processing to obtain the output $|y\rangle$ using the two inputs $|x_1\rangle$ and $|x_2\rangle$ is written $|y\rangle = k|(U|x_1\rangle, A|x_2\rangle)$. Given this learning mechanism "$k|(U|x_1\rangle, A|x_2\rangle)$", we define the *conditions enabling learning* (CEL) to be the conditions for the learning convergence fixed by the classical studies of the learning mechanism. For instance, if the learning mechanism is a conditioning mechanism, some pairs of unconditional and conditional stimuli must be presented more often than others, according to a particular probabilistic law and they must be linearly separable. This set of constraints constitutes the CEL. Let us consider a processing mechanism: in our example, either the unconditional pathway of a conditional learning mechanism, fig.4,a, or its conditional pathway after learning fig.4,b. The inputs of this processing mechanism can be split in two: the set of inputs the mechanism is specifically designed to process and the set of all the others inputs. The first of these two sets must be correctly processed by the mechanism whereas there are no guarantee for the correct processing of the second set. In figure 4,a, we have represented the space of couples of conditional and unconditional inputs of an abstract example of a conditioning mechanism. We have figured the sets of correctly processed inputs, the set of incorrectly processed inputs and the set of the ones for which some generalization properties of the considered mechanism can enable a correct processing. That is an idealized representation in which each subset is a compact region. In the same way, we have figured in fig.4,b the set of inputs correctly processed by the adaptive pathway after learning (a conditional pathway in our example). Note that, if the processing mechanism is an unconditional pathway, the correctly processed inputs are defined by the design of the architecture, and a subset of unconditional input is correctly processed, independently from the conditional inputs fig.4,a. Conversely, if the processing mechanism is an adaptive pathway, the correctly processed inputs are defined by the learning, and a subset of conditional input is correctly processed by the conditional pathway independently from the unconditional inputs fig.4,b.

Now, let consider the processing mechanism before and after learning. In our example, the processing mechanism is a conditioning mechanism (fig.5,a). We can represent the different sets of correctly processed inputs before and after learning, see fig.5,b vs fig.5,d. In this space of inputs (in our example, couples of conditional and unconditional inputs), we have represented the set of input determined by the CEL (fig.5,c): for instance, this region must be included in the intersection between the sets of inputs correctly processed by each pathway. Even if the CEL bound a relatively small region which will enable simplifications with the CSF, fig.5 shows that after learning the set of correctly processed couples of inputs should be much wider.

The CSF is settled on these CEL and uses them to transform a CS architecture in a simpler architecture, equivalent to the original one under the CEL. Hence the dynamical capabilities of the architecture can be determined and additional constraints for the convergence of learning can be fixed according to these dynamical capabilities. After the learning of the appropriate behavior under the CEL, the dynamics of the interactions with the environment is reduced to an attraction basin (stable perception state [Gaussier et al., 2004]) which corresponds to a sensory-motor invariant, accordingly to the embodiment and the constraints of the environment. Hence, two phases or states will be distinguished: a *before learning* state in which only the unconditional pathway of the architecture is efficient and an *after learning* state in which the adaptive processing mechanism should have converge in stable processing state (see fig.6).

According to the *before learning* and *after learning* states, we differentiate two types of simplifications [Gaussier et al., 2003]. Simplifications of the first type can be performed at any time, before or after

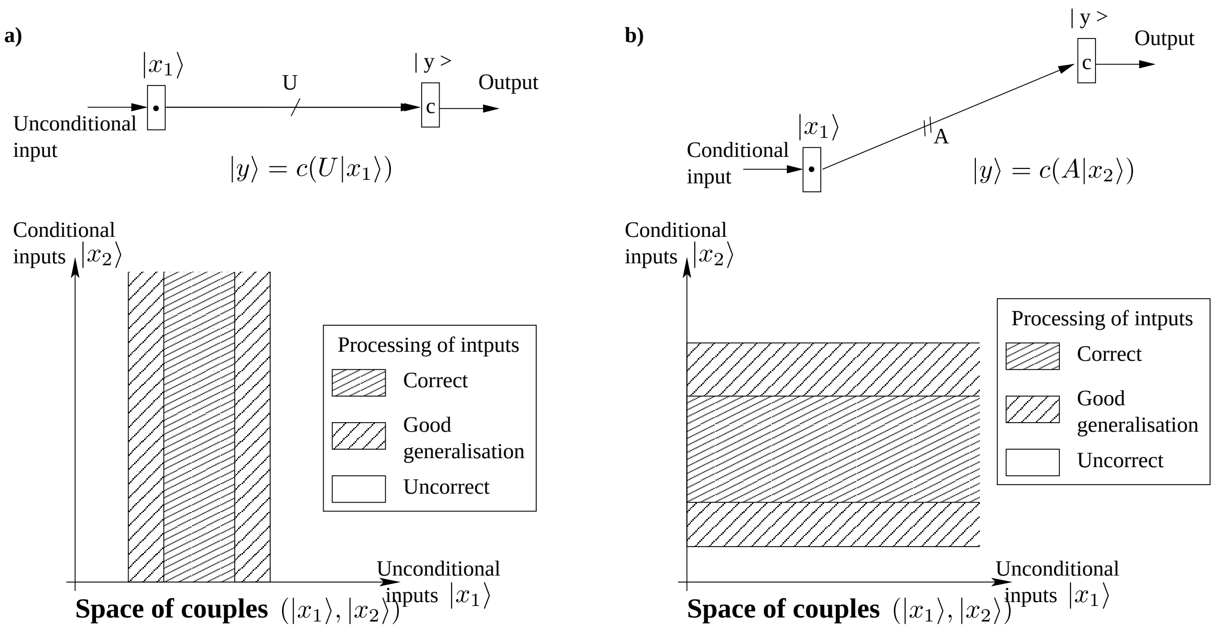

Figure 4: Given a fixed processing mechanism with a conditional (a) and an unconditional (b) input, the couples of inputs can be arranged in two sets: correctly vs incorrectly processed inputs (respectively tightly hatched vs white regions). In the set of incorrectly processed inputs, the inputs at the edges of the set of correctly processed inputs may be correctly processed thanks to generalization properties of the processing mechanism (hatched region). a) If the processing mechanism is an unconditional pathway, the correctly processed inputs are defined by the design of the architecture. A subset of unconditional input is correctly processed, independently from the conditional inputs. b) On the adaptive pathway, the correctly processed inputs are defined by the learning. A subset of conditional input can be correctly processed by the conditional pathway independently from the unconditional inputs.

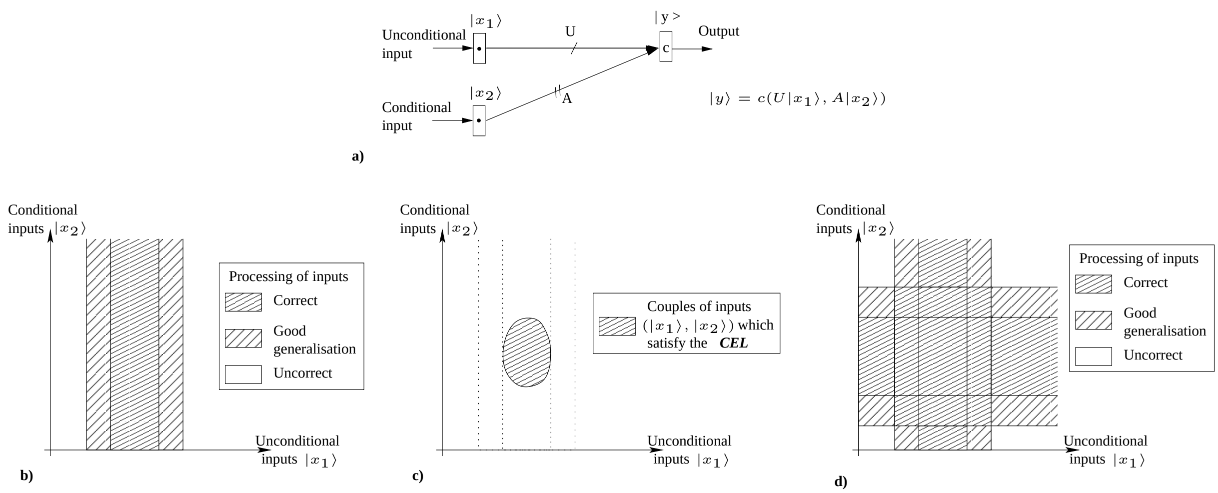

Figure 5: Given a processing mechanism (in our example the unconditional (a) and conditional (b) pathways of a prototypical conditioning mechanism), we have represented in an idealized way the couples of conditional and unconditional inputs. These couples of inputs are arranged in two sets: correctly vs incorrectly processed inputs. In the set of incorrectly processed inputs, the inputs at the edges of the set of correctly processed inputs may be correctly processed thanks to generalization properties of the processing mechanism (hatched region). a) Before learning, a conditioning mechanism is equivalent to its unconditional pathway. The only inputs which can be correctly processed are those which are "unconditional" and for which the processing mechanism of the unconditional pathway was dedicated to. b) Given a learning mechanism, the *conditions enabling learning* (CEL) determine the inputs which allow the convergence of learning. c) After learning, a generic conditioning mechanism is able to process either "conditional" inputs $|x_2\rangle$ or "unconditional" inputs $|x_1\rangle$, alone or together, and can generalize the learning to inputs out of the CEL.

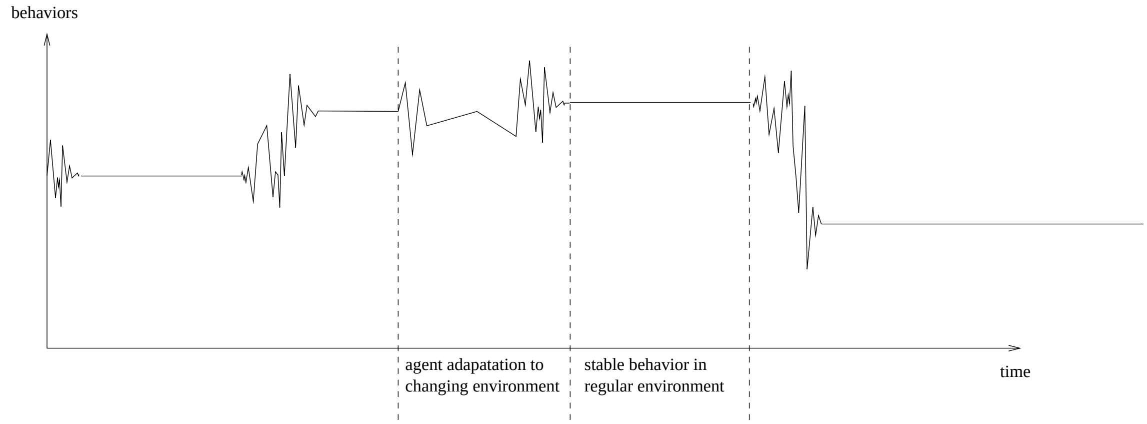

Figure 6: Intuitive representation of a "stable" behavior allowing the formal simplifications of a system.

learning, and under any conditions (no learning constraints), because the system keeps its learning properties unchanged. These are very limited simplification rules which only allow to reduce the number of elements in the architecture and to suppress trivial redundancies (i.e. simplification of a chain of reflex links). For instance a chain of two competitive groups connected by unconditional links can be written $c_1(U_1|c_2(U_2|x\rangle))$ and is equivalent to $c_1(U'|x\rangle)$ (see fig. 7. a). In the same way, when adaptive mechanism are connected in parallel, they can be written $c(A_1|x\rangle, A_2|x\rangle)$ and are equivalent to $c(A_3|x\rangle)$ (see figure 7.b).

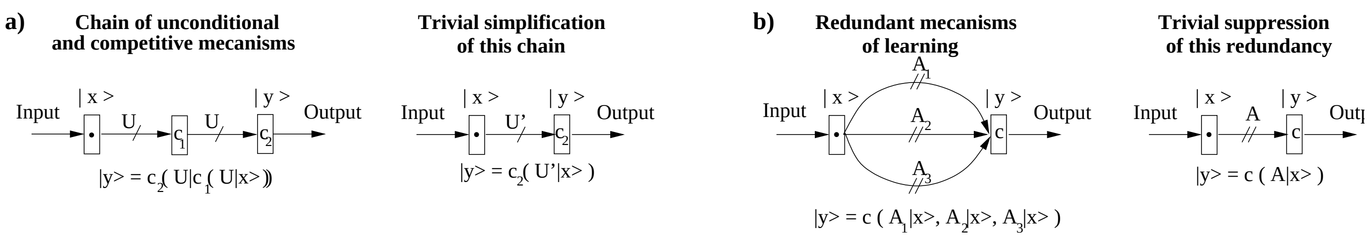

Figure 7: Trivial simplifications which can be performed at any time: they do not modify learning capabilities and dynamics of the architecture. a) simplification of a series of *Unconditional* links and competitive mechanisms: $c_2(U|c_1(U|x\rangle)) \equiv c_2(U'|x\rangle)$. b) simplification of a set of parallel *Adaptive* links between two boxes: $c(A_1|x\rangle, A_2|x\rangle, A_3|x\rangle) \equiv c(A_3|x\rangle)$.

Simplifications of the second type, *after learning* simplifications, are more ambitious simplifications which may imply changes in the architecture dynamics. They are much more useful simplifications to study the dynamics of a system since they point out additional constraints on the learning convergence. To be valid, these simplifications must be performed considering that the system is in a stable state of perception or interaction with its environment: the system must remain almost unchanged, the environment must be regular enough to avoid internal modifications of the robot's dynamics and the robot's behavior must be regular and predictable too. The simplifications are only valid during this *stable periods*. The learning must have converged and the environmental conditions must be restricted according to the CEL. These *after learning* simplifications focus on the dynamics of the system which have been acquired by the learning. They enable to study independently different parts of the architecture and their interactions with the environment. They allow to define additional constraints for the functioning of the original architecture (fig.8). Let us consider the space $\$$ of every possible dynamics of any architecture in an environment restricted to the CEL. The possible dynamics of the studied architecture under the CEL constitute a subspace $\$'$ of $\$$. The CSF aims to determine this subspace $\$'$. Determining $\$'$ is hard to perform while studying the architecture as a whole, but determining $\$'$ becomes much easier studying the different simplified architectures obtained using *after learning* simplifications rules. For instance, if the architecture is a prototypical conditioning mechanism, it will be simplified in two architectures (equivalent under the CEL): one with only the reflex pathway and the other with only the conditional pathway. These simplified architectures allow to determine the subspaces $\$_i$ associated to their functioning dynamics. When these simple architectures are put back together the global functioning dynamics is reduced to the intersection of the subspaces $\$_i$: $\$' = \cap \$_i$. Figure 8 shows an intuitive representation of the possible dynamics of the architecture under the CEL. A prototypical conditioning mechanism is embedded in an environment restricted to the CEL. Under these CEL and *after learning*, the conditioning mechanism is both equivalent to its reflex pathway and to its conditional pathway. As described just above, each one of the two equivalent architectures enables determining a subspace $\$_i$ of $\$$, and the intersection of these two subspace is $\$'$, which add to the CEL constraints on the possible dynamics of the whole architecture under the CEL: we define these additional constraints as the *constraints on the dynamics* for the learning convergence.

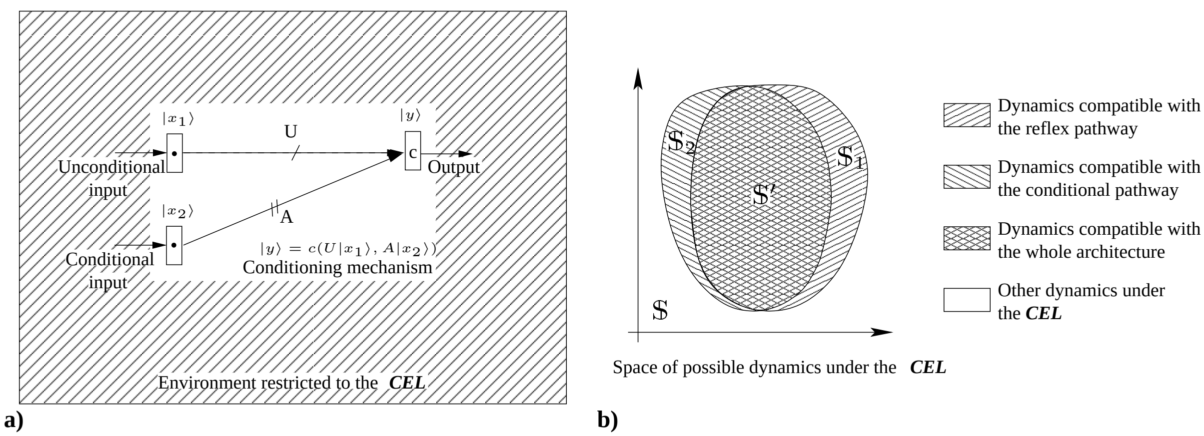

Figure 8: Boundary of the *constraints on the dynamics* for the convergence of learning. In a), the architecture is a prototypical conditioning mechanism embedded in the environment restricted to the CEL. b) represents, in an idealized way, the space of all the possible dynamics under the CEL, $\$$. Under the CEL, the conditioning mechanism is equivalent either to its reflex pathway or to its conditional pathway. Each simplified architecture determines a subspace of dynamics which enable interactions between the simplified architecture and the environment, for instance $\$_1$ for the reflex pathway and $\$_2$ for the conditional pathway. The intersection of these two subspaces is the subspace of dynamics $\$'$ which enables the interaction of the whole architecture with the environment.

The purpose of the CSF is to study the different capabilities of an architecture and among others,

what are the benefits of learning in terms of dynamics and behavioral attractors. We have seen how to use the CEL so as to fix additional *constraints on the dynamics* for the learning convergence. Let us consider that the learning has converged. During the learning phase, the *constraints on the dynamics* were necessarily satisfied, and thus, under the CEL, the behavioral attractors after learning are the same as the one fixed by the *constraints on the dynamics*. Thus we are able to generalize these dynamics to the environment not restricted to the CEL. We are able to determine which are the abilities acquired thanks to the learning, regarding a specific environment, or a task to learn (such as JA in our case). In our previous studies as well as in the present one, we alternately use these two point of view to find first the condition for a good learning (*constraints on the dynamics* added to the CEL) and next the solutions obtained after learning.
The choice of the formal simplification rules implies to explicit what is fundamental in the cognitive processes. These choices can and should be criticized by the scientific community (roboticians, psychologists, neurobiologists) in order to define a coherent set of fundamental equations taking into account a larger and more precise range of situations (i.e. defining new invariants linked to principles not taken into account in the current version of our formalism). The present work is an attempt to show such an approach is possible on an architecture developed by another lab and can already be fruitful, both from conceptual and practical points of view.

3 Formalization of the architecture for joint attention

In order to study the dynamics of the JAL architecture (fig. 9), it must first be written in a formal way. Next, the general schema and the equation representing the whole system (the human/robot interaction) could be deduced.

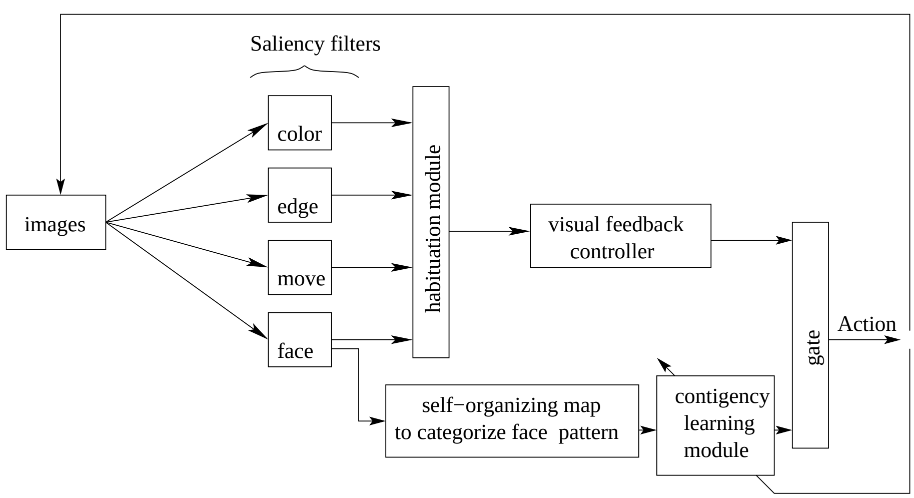

Figure 9: Reproduction of the JAL architecture which will be studied (derived from [Hosoda et al., 2004]). The arrows show the pathway of the signal trough the different modules.

Practically, it is easier to start the formalization analyzing the output of the robot. Let us sup-

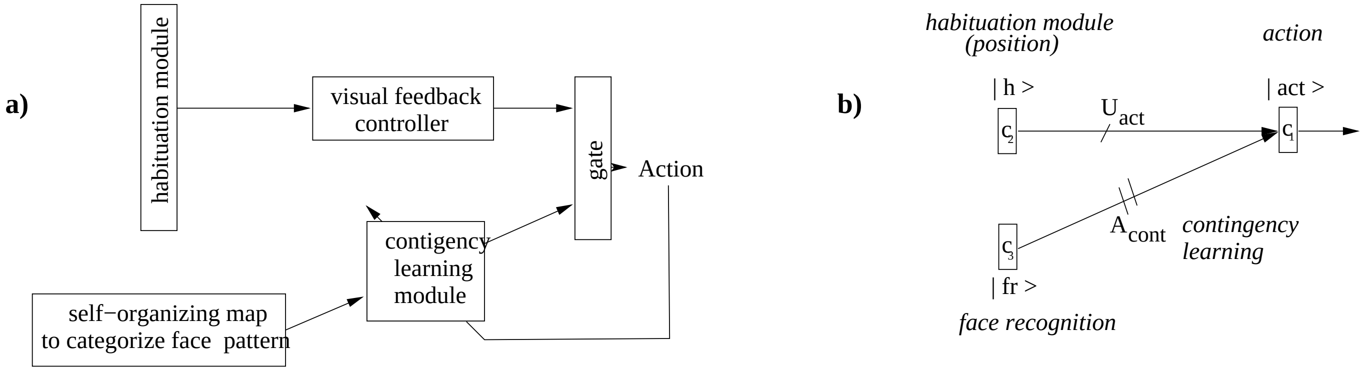

Figure 10: The *visual feedback controller* is an unconditional process and the *contingency learning module* is a conditional learning. Hence, the U_{act} matrix describes the reflex pathway and the adaptive matrix of contingency A_{cont} describes the conditional processing of the signal. At each step of the signal processing, the signals are written as vectors: $|h\rangle$, $|fr\rangle$ and $|act\rangle$. a) original writing of the architecture. b) formalization with the CSF.

pose the action of the robot is written as an output vector $|act\rangle$. This output is generated by the *gate* which processes a competition, noted c_1, between two signals. On one hand, the output vector $|h\rangle$ of the *habituation module* processed by the *visual feedback controller* (fig.10,a). By construction ([Hosoda et al., 2004]) this controller performs an unconditional treatment to obtain the action and is thus written as an unconditional matrix U_{act}. As a result, the first competitive signal is $U_{act}|h\rangle$. On the other hand the vector of *face recognition* $|fr\rangle$ is processed by the *contingency learning module* (fig. 10a). By construction this module learns to associate facial patterns $|fr\rangle$ with $U_{act}|h\rangle$ (the other signal in competition). In fact, it performs a conditional treatment learned by contingency and it is thus written as an adaptive matrix of contingency A_{cont}. The second competitive signal is $A_{cont}|fr\rangle$.

We can now draw the schema fig.10,b and to write the following equation 1 which describes the action $|act\rangle$ depending on both the habituation output $|h\rangle$ and the face recognition $|fr\rangle$:

$$|act\rangle = c_1\left(U_{act}|h\rangle,\ A_{cont}|fr\rangle\right) \tag{1}$$

The next step now is to formalize the elements before both the habituation output $|h\rangle$ and the face recognition $|fr\rangle$. Let consider the saliency filters which precede $|h\rangle$. The first rule of *Decomposition of a signal* of the CSF (see Appendix 8.1) shows that the different outputs of the filters can be written as a single vector of saliency $|s\rangle$ which concatenates the four filters. The *habituation module* generates an output $|h\rangle$ depending both on the *saliency filters* output $|s\rangle$ and on the preceding state of the *habituation module* itself (fig.11,a). This is neither an unconditional process, since the *habituation* changes through time, nor a learning process, since it follows a beforehand defined schema. We thus formalize this process by the product of the incoming signal $|s\rangle$ with an evolving matrix of habituation E_h. The evolution of the matrix E_h only depends on changes and novelty in the environment. At last, a competition mechanism c_2 makes the final decision $|h\rangle$. We thus have the equation 2 associated with fig.11,b:

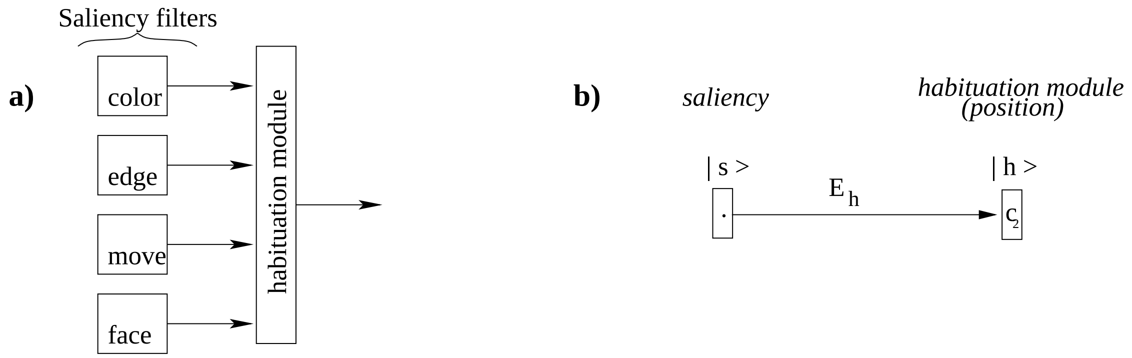

Figure 11: The four saliency filters outputs can be written as one vector of saliency$|s\rangle$ (section 8.1). The *habituation module* is an evolving (beforehand defined evolution) mechanism E_h which makes a choice of action $|h\rangle$ depending both on its input $|s\rangle$ and its own previous state. a) original writing of the architecture. b) formalization with the CSF.

$$|h\rangle = c_2(E_h|s\rangle) \tag{2}$$

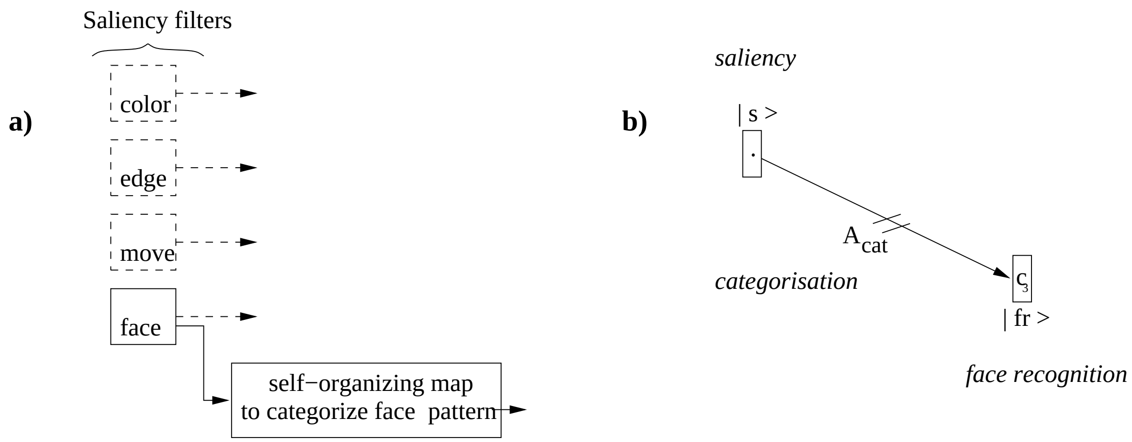

Figure 12: a) original writing of the saliency filters and face categorization. b) formalization with the CSF. The *self-organizing map* is a learning mechanism of categorization (learned matrix A_{cat}). The activity of the face recognition $|fr\rangle$ corresponds to the recognition of a face pattern $|f\rangle$. The face pattern $|f\rangle$ is one of the four outputs of the *saliency filters* part of $|s\rangle$. We show in the Appendix 8.1 that the process of A_{cat} on $|f\rangle$ is the same as the process of A_{cat} on $|s\rangle$.

The element preceding the face recognition $|fr\rangle$ is a *self-organizing map* which classifies face patterns $|f\rangle$ in a topological way depending on their reciprocal similarities (fig.12,a). It is a self-organized recognition mechanism which chooses a winner $|fr\rangle$ between different categories and reinforces the association between the stimulus and the chosen response. We will write it as an adaptive matrix for categorization A_{cat} applied to the face pattern vector $|f\rangle$ and followed by a competition mechanism c_3: $|fr\rangle = c_3(A_{cat}|f\rangle)$.

The face pattern $|f\rangle$ is one of the four outputs of the *saliency filters*. As it has been shown concerning the *habitation module*, these four signals can be written as a single signal of saliency $|s\rangle$. We show in the Appendix 8.1 (second rule of *Decomposition of signal*) that we can consider that the process of A_{cat} on $|f\rangle$ is the same as the process of A'_{cat} on $|s\rangle$ (implying changes without consequence on A_{cat}). We thus have the equation 3 associated to fig.12,b:

$$|fr\rangle = c_3(A_{cat}|f\rangle) = c_3(A'_{cat}|s\rangle) \tag{3}$$

For simplicity and since it does not involve any risky changes, we will write A_{cat} for A'_{cat}. Now, only one last part of the architecture remains to be formalized.

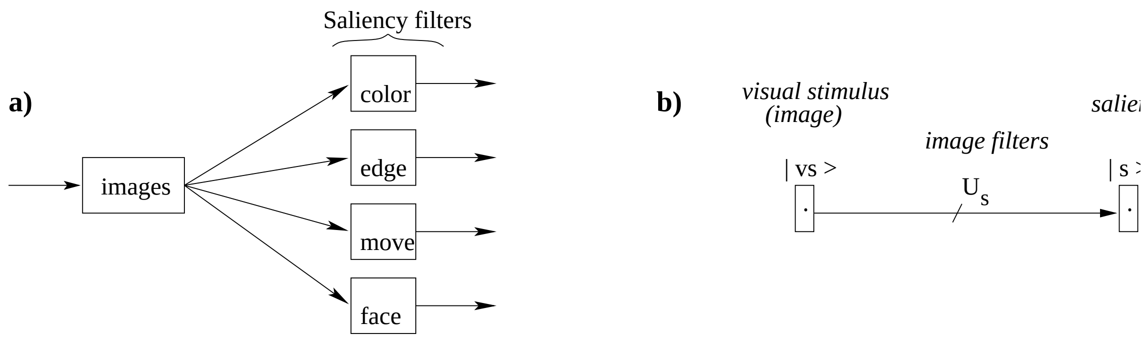

Figure 13: The *saliency filters* are four *Unconditional* filters $U_{s,i}$ ($i \in [1,4]$) which extract from input images four vectors $|s_i\rangle$ corresponding to four salient properties of the visual stimuli. We show in the appendix 8.1 that this four processes can be written as a single process U_s giving a single output vectors of saliency properties $|s\rangle$. a) original writing of the architecture. b) formalization with the CSF.

The *saliency filters* extract relevant properties (color, edges, move and face) from the visual stimulus $|vs\rangle$ (left schema of fig.13). Each filter processes an unconditional linear treatment $U_{s,i}$ ($i \in [1,4]$) to obtain a salient feature $|s_i\rangle$. We show in the appendix 8.1 that we can replace the matrices U_i by a single unconditional matrix U and the $|s_i\rangle$ by a single signal of saliency $|s\rangle$ (see third rule of *Decomposition of signal*). We obtain the equation 4 associated with schema of fig. 13:

$$|s\rangle = U|vs\rangle \tag{4}$$

Now, we have formalized each part of the original architecture. We can deduce the equation representing the whole architecture starting from the action vector $|act\rangle$. We replace the habituation output $|h\rangle$ and the face recognition $|fr\rangle$ in the equation 1 using equations 2 and 3:

$$\begin{aligned} |act\rangle &= c_1\left(U_{act}|h\rangle,\ A_{cont}|fr\rangle\right) \\ &= c_1\left(U_{act}|c_2(E_h|s\rangle),\ A_{cont}|c_3(A_{cat}|s\rangle)\right) \end{aligned} \tag{5}$$

We end replacing the saliency vector $|s\rangle$ in equation 5 using equation 4:

$$|act\rangle \quad = \quad c_1(U_{act}|c_2(E_h|U|vs\rangle),\; A_{cont}|c_3(A_{cat}|U|vs\rangle)) \tag{6}$$

We can here notice that our formalism has allowed to obtain a very compact representation of the whole initial JAL architecture, from the visual stimuli to the head movement control, by a single equation (equation 6).
We can also draw the schema fig.14, which represents the whole architecture, giving an overview of every of its flows of information and of the way they are processed.

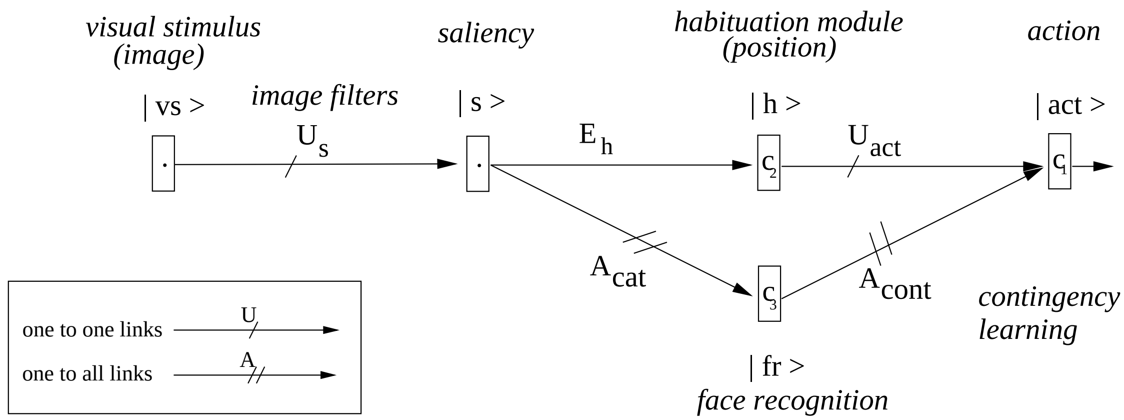

Figure 14: Schema of the whole architecture. The symbols on the different arrows represent the different matrices and operators characterizing the processing of the signal. The vectors, which represent the signal in different points of the architecture, are written on the top or on the bottom of the different operator boxes (the letters inside the boxes represent the operators characterizing the combination of the signals and the matrices, and the learning).

The embodied JAL architecture extracts two independent signals from the visual stimuli $|vs\rangle$: objects features and face pattern. We assume that the visual stimulus $|vs\rangle$ can be written as two independent stimuli: the objects on the table and the face pattern of the human interacting with the robot. The visual stimulus can be thus written $|vs\rangle = |obj\rangle + \;|face\rangle$ with $|obj\rangle$ the vector representing the objects (including the other agent face) and with $|face\rangle$ the vector coding for the face and its gaze direction. The consequence of such a decision is that we assume the independence of the two pathways, *Adaptive* and *Unconditional* (see fig.15).

Thus, the equation 5 can also be rewritten as the equation 7 where action $|act1\rangle$ is expressed according to the objects salient features $|s\rangle$ and the face pattern $|f\rangle$. $|s\rangle$ and $|f\rangle$ are extracted from the environment: the objects (including the human face) $|obj\rangle$ and the face $|face\rangle$ by unconditional matrices U'_s and U_f respectively.

$$\begin{aligned} |act1\rangle \quad &= \quad c_1(U_{act}|c_2(E_h|s\rangle),\; A_{cont}|c_3(A_{cat}|f\rangle)) \\ &= \quad c_1(U_{act}|c_2(E_h \cdot U'_s|obj\rangle),\; A_{cont}|c_3(A_{cat} \cdot U_f|face_2\rangle)) \end{aligned} \tag{7}$$

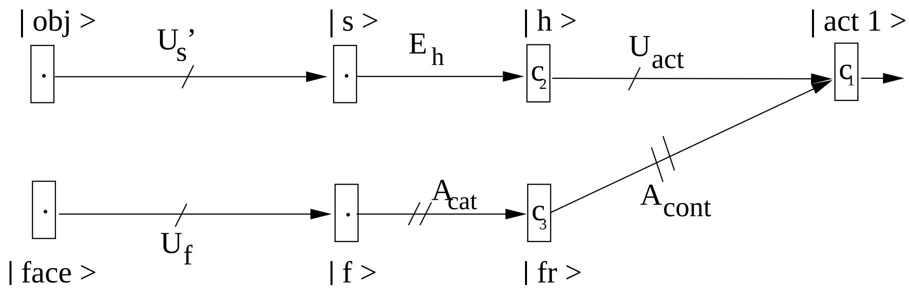

Figure 15: The JAL architecture is rewritten with two pathways processing independently objects and face features. ($|obj\rangle$ is the vector describing objects, U_s' is the unconditional saliency matrix, h is for habituation, f is for face, A_{cat} is for the adaptive categorization matrix, fr is for face recognition, *cont* is for contingency and *act* is for action).

The formalization of the architecture is almost complete but one crucial point remains. In any architecture, two distant points on the same information flow are separated by a certain amount of computation (processes): For instance $|obj\rangle$ and $|act1\rangle$ are both on the *Unconditional* pathway and are separated by the *saliency filters* and the *habituation module*. This amount of processes takes a certain time of processing. In the present architecture, the *temporal distance* separating the action from the stimulus can be defined according to either the *Unconditional* pathway or the *Adaptive* pathway. The building of the architecture (software choices of synchronous architecture), which computes an action using either the *habituation module* decision or the *face recognition* pathway, implies that the *temporal distance* between $|obj\rangle$ and $|act1\rangle$ on one hand and between $|face_2\rangle$ and $|act1\rangle$ on the other hand, be the same. This *temporal distance* will be noted Δt_1.

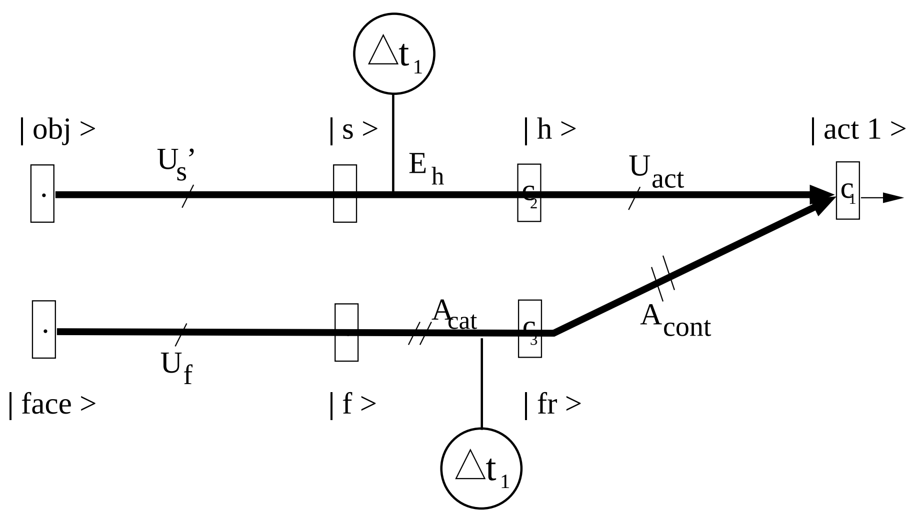

Figure 16: Architecture written with *temporal distances* between action and stimulus. The architecture design constraints both pathways (*Unconditional* and *Adaptive*) to be synchronized. A single *temporal distance* Δt_1 is defined.

The action, $|act1(t)\rangle$, at time "t" depends on the stimuli, $|obj(t-\Delta t_1)\rangle$ and $|face_2(t-\Delta t_1)\rangle$ at time

"$t - \Delta t_1$". Adding this formalization of time, the equation 7 should be written as follows:

$$|act1(t)\rangle \quad = \quad c_1(U_{act}|c_2(E_h \cdot U'_s|obj(t-\Delta t_1)\rangle),\ A_{cont}|c_3(A_{cat} \cdot U_f|face_2(t-\Delta t_1)\rangle)\rangle) \tag{8}$$

This equation describes formally both the complete architecture and its internal dynamics. Now, to study the architecture capabilities, the interaction with the environment and the human must also be formalized in order to obtain a global set of equations which will represent the potential dynamics of the system agent/environment.

4 The formalized agent immersed in its environment

To study the capabilities of a robot using the JAL architecture, we will formally immerse it in an environment corresponding to the original experimental setup. Indeed, the JAL architecture has been rewritten as a classical input/output system, forgetting the importance of the dynamical interactions as in the classical AI paradigm. The external links from the outputs to the inputs has to be formalized so as to close the *Sensory-Motor* loop. An *Agent2* which demonstrates face features and interacts with the robot (*Agent1*) is thus introduced in the *Sensory-Motor* loop. In the JAL experimental setup, *Agent2* is a human. He or she processes his or her actions using both the objects features and the gaze direction of the robot. These stimuli are equivalent to those the robot can use: objects features and face pattern of the human. At this point, it is interesting to notice that the only association and learning point between the independent pathways is the output of the competition mechanism c_1. The operator c_1 controls the A_{cont} learning of contingencies between the two pathways. Thus, to enable convergence of learning, the two signals converging to c_1 must be correlated. Inside the architecture, these two signals are totally independent because of the independence of the *Unconditional* and the *Adaptive* pathways. Hence, the two signals must be correlated outside the architecture, i.e. in the objects/human environment. The vector $|face\rangle$ represents the *Agent2*'s face pattern. In the studied experimental setup, it characterizes the *Agent2*'s gaze direction. Considering that the vector $|face\rangle$ depends on the *Agent2*, the correlations between the $|obj\rangle$ and$|face\rangle$ incoming vectors depend on the *Agent2*'s behavior. For instance, if *Agent2* gazes at one of the objects, he demonstrates a face pattern $|face\rangle$ correlated with the object location $|obj\rangle$. An operator ξ is introduced to characterize how *Agent2* links *Agent1*'s gaze direction and the objects location through its own gaze direction. Finally, the interaction between both agents is represented by the global architecture of figure 17.

At a given time t, the outputs of the human caregiver, *Agent2*, depend on the stimuli occurring at $t-\Delta t_2$: $|obj(t-\Delta t_2)\rangle$ and $|act_i(t-\Delta t_2)\rangle$. Δt_2 represents the processing time of *Agent2* (from its inputs to its outputs see fig. 18).

Hence, the outputs of ξ at time t, $|act2(t)\rangle$, are written as follow in equation 9:

$$|act2(t)\rangle = \xi(|obj(t-\Delta t_2)\rangle, |act(t-\Delta t_2)\rangle) \tag{9}$$

and by the same way, for $t - \Delta t_1$ we have $|act2(t-\Delta t_1)\rangle = \xi(|obj(t-\Delta t_1-\Delta t_2)\rangle, |act(t-\Delta t_1-\Delta t_2)\rangle)$ Defining $\Delta t_3 = \Delta t_1 + \Delta t_2$ (duration of a complete interaction loop), the equation becomes equation

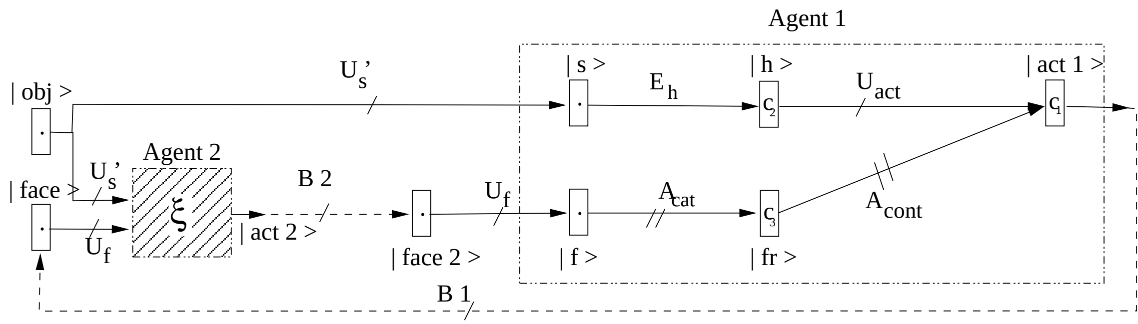

Figure 17: *Agent2*'s behavior depends on both objects and *Agent1* (the robot) behavior, through the operator ξ. $|face_i\rangle = B_i|act_i\rangle$ and the matrix B_i represents the effect of the gaze direction (action) on the image of the face.

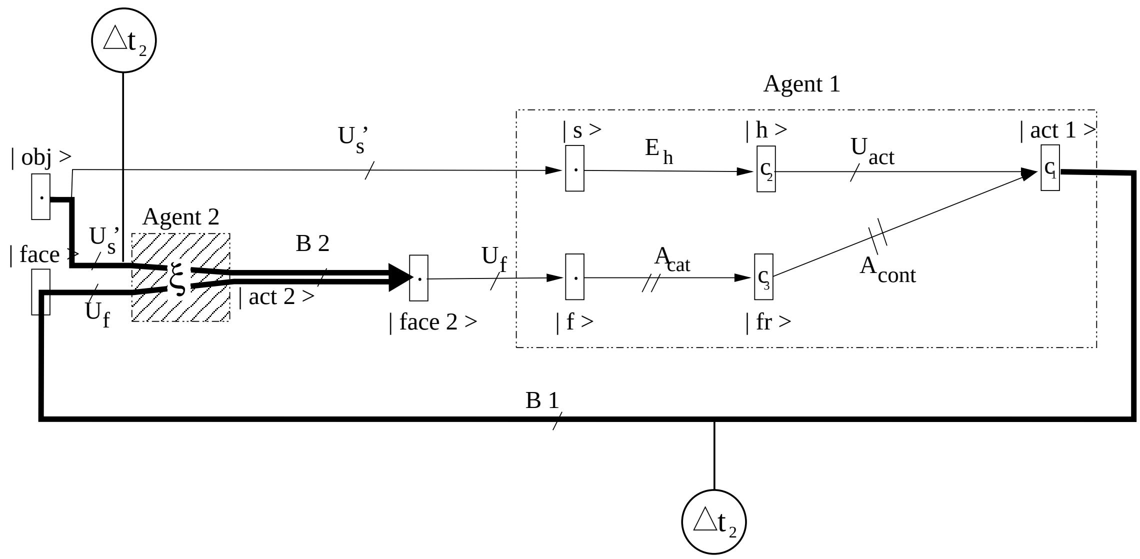

Figure 18: The JAL architecture written with two pathways processing independently objects and face features. $|obj\rangle$ is the vector describing objects, U'_s is the unconditional saliency matrix, in the same way $|h\rangle$ is for habituation, $|f\rangle$ is for face, A_{cat} is for the adaptive categorization matrix of facial information, $|fr\rangle$ is for face recognition, A_{cont} is the matrix used for contingency learning, $|act\rangle$ is for action.

10:

$$|act2(t-\Delta t_1)\rangle = \xi(|obj(t-\Delta t_3)\rangle, |act(t-\Delta t_3)\rangle) \tag{10}$$

The equation 8 which represents the output gaze control of the robot (*Agent1*) must be slightly modified to take into account the human caregiver's behavior ξ (*Agent2*):

$$\begin{aligned}
|act1(t)\rangle &= c_1(U_{act}|c_2(E_h \cdot U'_s|obj(t-\Delta t_1)\rangle),\ A_{cont}|c_3(A_{cat} \cdot U_f|face_2(t-\Delta t_1)\rangle)\rangle) \\
&= c_1(U_{act}|c_2(E_h \cdot U'_s|obj(t-\Delta t_1)\rangle),\ A_{cont}|c_3(A_{cat} \cdot U_f|B_2|act2(t-\Delta t_1)\rangle)\rangle) \\
&= c_1(U_{act}|c_2(E_h \cdot U'_s|obj(t-\Delta t_1)\rangle),\ A_{cont}|c_3(A_{cat} \cdot U_f|B_2|\xi(U_f|act1(t-\Delta t_3)\rangle, U'_s|obj(t-\Delta t_3)\rangle)\rangle)\rangle)
\end{aligned} \tag{11}$$

Considering that if the input of the operator ξ can be written as the product of an *unconditional matrix* U by a vector $|v\rangle$ then $\xi(U|v\rangle) = \xi'(|v\rangle)$ where ξ' is the compound function $\xi \circ U$ and thus have the same learning and dynamic properties as ξ , $\xi(U_f|act1\rangle, U'_s|obj\rangle)$ will be written $\xi'(|act1\rangle, |obj\rangle)$. We also have the matrices product $A_{cat} \cdot U_f$ which can be written A'_{cat}. Equation 11 becomes equation 12 (see Appendix 8.2):

$$|act1(t)\rangle \quad = \quad c_1(U_{act}|c_2(E_h \cdot U'_s|obj(t-\Delta t_1)\rangle\rangle),\ A_{cont}|c_3(A'_{cat} \cdot \xi'(|act1(t-\Delta t_3)\rangle, |obj(t-\Delta t_3)\rangle\rangle\rangle)) \tag{12}$$

Both figure 17 and equation 12 shows the crucial role of the c_1 operator and the A_{cont} matrix as they enable the architecture to perform the recognition of the gaze direction and its association with the correct robot actions (to look at the same objects the human caregiver is looking at). Figures 16 and 18 and equation 12 describe the temporal constraints of the experimental setup. To sum up, the dynamics of the interaction loop between the robot and its environment has been formalized. The human/robot system constitutes a dynamical system which must achieve joint attention. We will try now to study which are the constraints to achieve this dynamical behavior.

5 Learning constraints

In this section, we will demonstrate that the correct achievement of JA by the robot implies a strong cooperation of the caregiver. For instance, we will show that the learning cannot converge if the robot faces another robot controlled by the same architecture (even if this other robot has already learned to perform JA). To solve this problem, we will show there is a need to change the internal dynamics of the robot architecture.

In order to study the JAL ability of the whole system robot/caregiver, we will first determine the *constraints on the dynamics* enabling learning and then, we will study what are the interaction capabilities acquired by this learning. In section 2, two aspects of the learning constraints were distinguished. First there are the *conditions enabling learning* (CEL) due to the learning mechanism and its implementation. The CEL depend on the building of the architecture, are determined by the classic studies of learning convergence and are not addressed by the CSF. Second there are the *constraints on the dynamics* of the whole system *architecture-environment*. The respect of these constraints is a necessary condition to allow interactions between the architecture and its environment. The CSF will enable to characterize these constraints. Now we will suppose that the environment is restricted to the *conditions enabling learnings* (see fig19).

Under the CEL, the "*after learning* simplifications" of the architecture can be performed. These simplifications will lead to two different architectures (one for the *unconditional* pathway and another for the *adaptive* pathway). We will be able to study what are the possible dynamics of each pathway. In

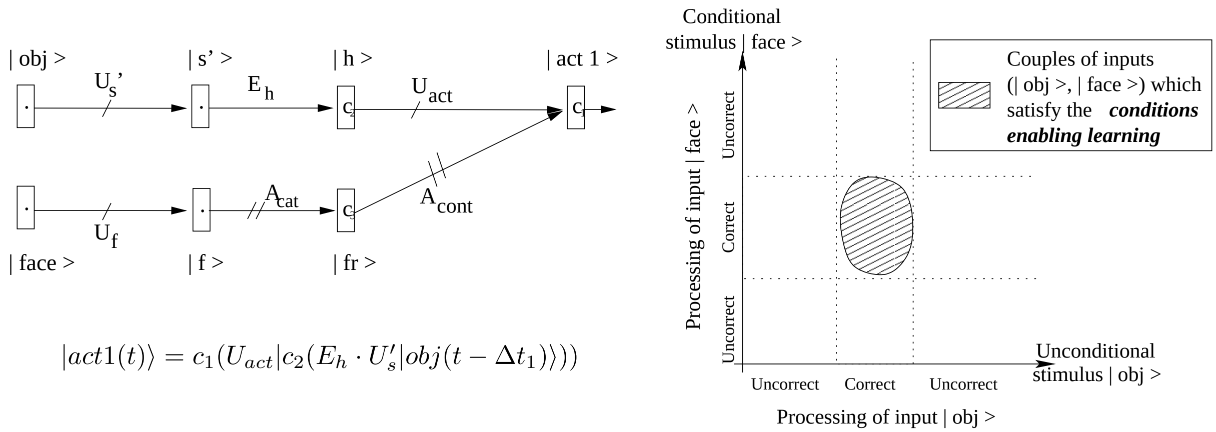

Figure 19: Intuitive representation of the *conditions enabling learning* (CEL)

the environment restricted to the CEL, the learning may converge if both *Unconditional* and *Adaptive* pathways compute the same output: the two pathways must be equivalent. Thus, after learning and in the environment restricted to the CEL, the term $U_{act}|c_2(E_h \cdot U_s'|obj(t-\Delta t_1)\rangle\rangle$ representing the *unconditional pathway* of the robot architecture (Equ.12) should compute the same output $|act1\rangle$ as the term $A_{cont}|c_3(A'_{cat} \cdot \xi'(|act1(t-\Delta t_3)\rangle, |obj(t-\Delta t_3)\rangle\rangle)\rangle$ representing the *adaptive pathways* ($\Delta t_3 = \Delta t_1 + \Delta t_2$ is the duration of a whole interaction loop and Δt_1 is the processing duration within the robot). Thus after learning and under the CEL, Equ.12 must be equivalent on one hand to equ.13 and on the other hand to equ. 14 ($|act1\rangle$ computed by the adaptive pathway):

$$|act1(t)\rangle = c_1(U_{act}|c_2(E_h \cdot U_s'|obj(t-\Delta t_1)\rangle\rangle) \tag{13}$$

$$|act1(t)\rangle = c_1(\ A_{cont}|c_3(A'_{cat} \cdot \xi_t'(|act1(t-\Delta t_3)\rangle, |obj(t-\Delta t_3)\rangle\rangle\rangle)) \tag{14}$$

Considering that $E_h \cdot U_s'$ and U_{act} are two matrices and c_1 and c_2 are competition mechanisms, we can simplify a chain of Unconditional links and competitive mechanisms. A matrix U_s'' does exist which verifies $c_1|U_s'' = c_1(U_s|c_2(E_h \cdot U_s'))$ (U_s'' unconditional because both U_{act} and $E_h \cdot U_s'$ are unconditional). The simplification of the reflex chain does not influence the system dynamics (see section 2 $c_1(U|c_2(U|x\rangle)) \equiv c_1(U'|x\rangle)$ and Appendix 8.2 for the details). Equation 13 becomes equation 15:

$$|act1(t)\rangle = c_1(U_s''|obj(t-\Delta t_1)\rangle\rangle) \tag{15}$$

Considering that, on one side, A_{cont} and A'_{cat} are two adaptive matrices and, on the other side, c_1 and c_3 are competition mechanisms, we can also simplify the chain of *Adaptive* mechanisms. A matrix A'' does exist which verifies $c_1(A''|x\rangle) = c_1(A_{cont}|c_3(A'_{cat}|x\rangle))$ (A'' adaptive because A_{cont} and A'_{cat} are adaptive). The simplification of this chain can influence the real adaptation capacity of the system, but theoretically this should not influence the system dynamics (see the Appendix 8.2 for the details). Equation 14 becomes equation 16:

$$|act1(t)\rangle = c_1(A'' \cdot \xi_t'[|act1(t-\Delta t_3)\rangle, |obj(t-\Delta t_3)\rangle]) \tag{16}$$

Finally, the *constraints on the dynamics* of each pathway (equations 15 and 16) are satisfied if both equation of the set 17 are verified (i.e. the two pathways of the architecture, *Unconditional* and *Adaptive*, produce the same result $|act1\rangle$):

$$\left\{ \begin{array}{lcl} |act1(t)\rangle & = & c_1(A''|\xi'(|act1(t-\Delta t_3)\rangle, |obj(t-\Delta t_3)\rangle)\rangle) \\ |act1(t)\rangle & = & c_1(U_s''|obj(t-\Delta t_1)\rangle) \end{array} \right. \tag{17}$$

The set of equation 17 ensure the coherence of the whole architecture dynamics after learning, under the CEL. In the environment restricted to the CEL, the learning may converge if both *Unconditional* and *Adaptive* pathways compute the same output: the two pathways are equivalent. After learning and in the environment restricted to the CEL, according to the set of equations 17 we must have $c_1(U_s''|obj(t-\Delta t_1)\rangle) = c_1(A'' \cdot \xi'(|act1(t-\Delta t_3)\rangle, |obj(t-\Delta t_3)\rangle))$. These constraints are necessary to allow the learning convergence. This proves and means that the gaze direction of *Agent2* $\xi'(|act1\rangle, |obj\rangle)$ and the $|obj\rangle$ stimulus must be correlated. Hence, to enable *Agent1* to learn associations between gaze direction and actions, the *Agent2*'s actions must be related either to objects locations or to *Agent1*'s actions or to both. These constraints will be now the ground for the study of the architecture dynamics depending on *Agent2*'s behavior.

6 Which dynamics enable convergence of learning?

We will study now which dynamics enable the convergence of learning. The operator ξ characterizes the human caregiver's behavior (*Agent2*) and its ability to link at will the robot's gaze direction (*Agent1*) with the objects position through its own gaze direction.

If ξ function (Agent2's behavior) only depends on objects, the *Agent2* must gaze at the objects without relating his/her gaze to *Agent1*'s actions (for instance choosing randomly target object). Objects' positions are the only link between *Agent1*'s actions and *Agent2*'s gaze direction. We assume here that the *Agent2*'s behavior is totally independent from the *Agent1*'s actions $|act1\rangle$ (robot actions, see fig.20).

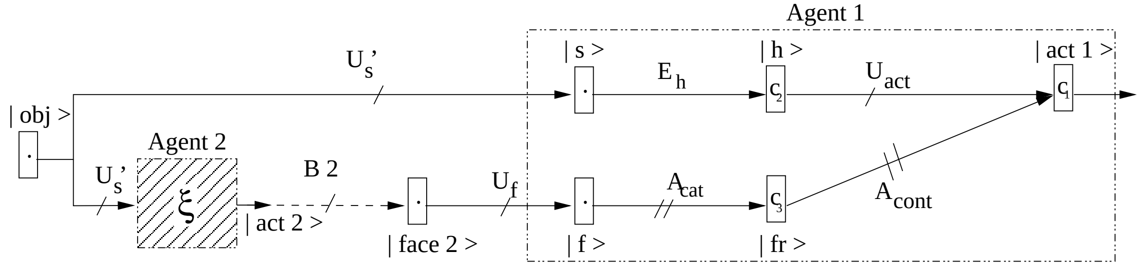

Figure 20: The agent facing the robot only sees the objects.

Thus, in this case, ξ' (the function computing the action vector $|act2\rangle$ of *Agent2*) is not anymore a function of both the robot's actions $|act(t-\Delta t_3)\rangle$ and the objects $|obj(t-\Delta t_3)\rangle$ but it is only a function of the objects location $|obj(t-\Delta t_3)\rangle$: $|act2\rangle = \xi'|obj\rangle$. The set of equations 17 becomes:

$$\left\{\begin{array}{rcl} |act1(t)\rangle & = & c_1(A''|\xi'(|obj(t-\Delta t_3)\rangle))) \\ |act1(t)\rangle & = & c_1(U''_s|obj(t-\Delta t_1)\rangle) \end{array}\right. \tag{18}$$

As long as the objects are not moved, whatever Δt is, the objects vector is constant and thus is the same at time t and time $t+\Delta t$: $|obj(t)\rangle = |obj(t+\Delta t)\rangle$. Hence, given a fixed location of the objects the system of equations becomes:

$$\left\{\begin{array}{rcl} |act1(t)\rangle & = & c_1(A''|\xi'(|obj(t)\rangle))) \\ |act1(t)\rangle & = & c_1(U''_s|obj(t)\rangle) \end{array}\right. \tag{19}$$

We can deduce:

$$c_1(A''|\xi'(|obj\rangle)) = c_1(U''_s|obj\rangle) \tag{20}$$

The processing of the objects $|obj\rangle$ within the robot must be the same as the processing of the objects by both the *Agent2* and the *Adaptive* pathway of the robot. The equation 20 sums up the conditions enabling learning convergence of the whole system robot/human/obj when the human does not take into account the robot gaze to perform his actions. These *constraints on the dynamics* are redundant with the classical *conditions enabling learning* (CEL) given by the learning mechanism itself: if the stimuli are correlated with each other, the learning converges. Given these CEL, the learning of associations between $|face2\rangle$ and $|act1\rangle$ should be possible. After learning, the associations learned will verify equ.20. That means, after learning convergence, the actions of the robot processed using the human gaze direction $|face\rangle$ (*Adaptive* pathway) verify the equation 21:

$$|act1(t)\rangle = c_1(A''|face2(t-\Delta t_1)\rangle)) = c_1(\alpha U''_s|obj\rangle). \tag{21}$$

The robot actions $|act1(t)\rangle$ only depends on the objects location: $|face\rangle$ is associated to the objects locations. If there was only one object, the robot looks where the human looks and in this trivial case, JA is performed, but if there were several objects during learning, thus, after learning the robot looks in the direction of one of the objects, not necessarily the one the human is looking at. This behavior is not relevant for JA since. To sum up, if the objects location does not change the learning of the adaptive matrix A'' converges but does not allow the robot to achieve JA. Finally, the equation 21 points out the fact that if, in the experimental setup, both the objects do not move and the caregiver only takes into account the objects, the system does not contains the internal resources enabling the achievement of the joint attention. Hosoda et al. show that this lack can be fulfilled by an external intervention. They show in [Hosoda et al., 2004] that if the objects are moved (action, out of the system robot/objects/human) the relevant associations will be the more reinforced by learning and thus the JAL will be achieved. The proof of such a result is not the goal of the CSF since this proof must take into account the way the objects are moved (it is a classical machine learning problem).

If ξ function (caregiver's behavior) only depends on the robot's actions $|act1\rangle$, it means the *Agent2* does not take into account the objects on the table and only considers *Agent1*'s gaze direction (see figure 21 for the corresponding schema).

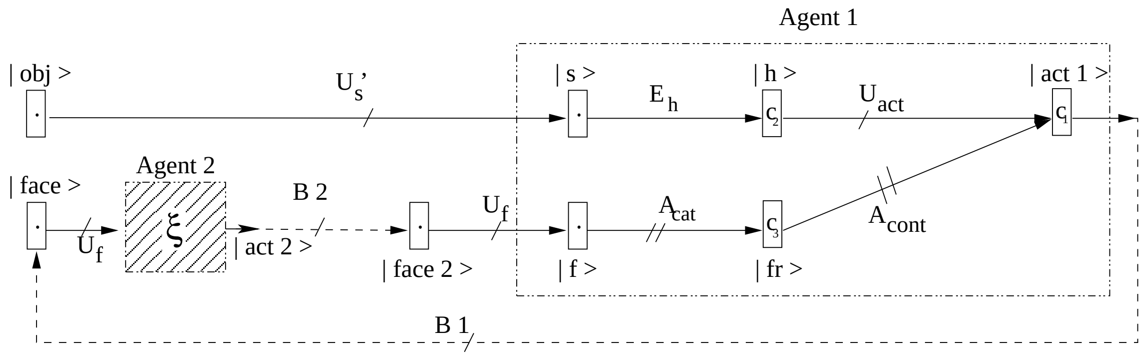

Figure 21: The human (agent facing the robot) does not matter about the objects on the table ($|obj\rangle$) and only takes into account the gaze direction of the robot ($|act1\rangle$).

ξ' (the function computing the actions vector of *Agent2* $|act2\rangle$) is not anymore a function of both the robot's actions $|act1\rangle$ and the objects $|obj\rangle$ but is only a function of $|act1\rangle$: $|act2\rangle = \xi'|act1\rangle$ Hence system 17 becomes equivalent to:

$$\begin{cases} |act1(t)\rangle &= c_1(A''|\xi'(|act1(t-\Delta t_3)\rangle)) \\ |act1(t)\rangle &= c_1(U_s''|obj(t-\Delta t_1)\rangle) \end{cases} \tag{22}$$

While the objects location does not change, $|obj(t-\Delta t_1\rangle = |obj(t)\rangle$, we have the set of equations 23:

$$\begin{cases} |act1(t)\rangle &= c_1(A''|\xi'[|act1(t-\Delta t_3)\rangle]) \\ |act1(t)\rangle &= c_1(U_s''|obj\rangle) \end{cases} \tag{23}$$

In equations 23, the adaptive matrix A'' can converge if $|act1(t)\rangle = |act1(t-\Delta t_3)\rangle$ ($\Delta t_3 = \Delta t_1 + \Delta t_2$ is the duration of an interaction loop). Conversely, if there is no relation between $|act1(t)\rangle$ and $|act1(t-\Delta t_3)\rangle$, the matrix A'' verifying equ.23 does not exist. That allows to define the *constraints on the dynamics* of equation 24 which enable the learning convergence.

$$\forall\ t,\ |act1(t)\rangle \ = \ |act1(t-\Delta t_3)\rangle \tag{24}$$

Equation 24 shows that the only thing the *Agent1* could learn is a temporal relation between its action at time t and at time $t-\Delta t_3$. In equation 10, we have defined the duration of an interaction loop Δt_3 as $\Delta t_3 = \Delta t_1 + \Delta t_2$ with Δt_1 the processing time within the robot and Δt_2 the processing time within the environment. Thus, the temporal relation Δt_3 that *Agent1* could learn can be decomposed in two parts. On one hand, Δt_1 is due to *Agent1* own architecture and it is thus fixed by the design of the architecture. On the other hand, Δt_2 is due to *Agent2* behavior and thus cannot be controlled by the robot: Δt_2 can take several values depending on *Agent2*'s behavior. If *Agent2* is a mirror, its duration of processing is null, $\Delta t_2 = 0$, and then Δt_3 can be written as follows (equ. 25):

$$\Delta t_3 = \Delta t_1 + \Delta t_2 = \Delta t_1 + 0 = \Delta t_1 \tag{25}$$

The *constraints on the dynamics* of the robot action $|act1\rangle$ (equ.24) becomes equation 26:

$$\forall\ t,\ |act1(t)\rangle\ =\ |act1(t-\Delta t_1)\rangle \tag{26}$$

To satisfy this equation, the robot must demonstrate a cyclic activity where its actions at time t is the same as its actions at time $t-\Delta t_1$: $|act1(t)\rangle = |act1(t-\Delta t_1)\rangle$. Here, nothing in the architecture enables us to assume that *Agent1* has this cyclic activity. The learning cannot converge. If *Agent2* is an imitator with an architecture quite similar to *Agent1*'s, the processing durations of both agents are quiet the same, $\Delta t_2 \simeq \Delta t_1$ and then the whole interaction loop duration Δt_3 is given by equation 27:

$$\Delta t_3 = \Delta t_1 + \Delta t_2 = \Delta t_1 + \Delta t_1 = 2\cdot\Delta t_1 \tag{27}$$

And the *constraints on the dynamics* of the robot action $|act1\rangle$ (equ.24) becomes equation 28:

$$\forall\ t,\ |act1(t)\rangle\ =\ |act1(t-2\cdot\Delta t_1)\rangle \tag{28}$$

To satisfy this new equation, the robot must demonstrate a cyclic activity where its actions at time t is the same as its actions at time $t-2\cdot\Delta t_1$: $|act1(t)\rangle = |act1(t-2\cdot\Delta t_1)\rangle$. Here again, nothing in the architecture enables us to assume that *Agent1* has this cyclic activity. The learning cannot converge. If *Agent2* is a human imitator, we can assume that his/her processing speed is such higher than the robot's one that it could be neglected ($\Delta t_2 \ll \Delta t_1$). Hence $\Delta t_2 \simeq 0$ and the duration of an interaction loop Δt_3 becomes $\Delta t_3 = \Delta t_1 + \Delta t_2 \simeq \Delta t_1 + 0 = \Delta t_1$. That is the same situation as in equation 25 when the robot faces a mirror: the robot learning cannot converge.
At last, if the *Agent2* demonstrates a much more complex behavior consisting in anticipating the *Agent1*'s action, then $\Delta t_2 \simeq\ -\Delta t_1$. The duration of the interaction loop Δt_3 is given by equation 29:

$$\Delta t_3 = \Delta t_1 + \Delta t_2 \simeq \Delta t_1 - \Delta t_1 = 0 \tag{29}$$

The *constraints on the dynamics* of the robot's action $|act1\rangle$ (equ. 24) becomes equation 30:

$$\forall\ t,\ |act1(t)\rangle\ =\ |act1(t)\rangle \tag{30}$$

This condition is always satisfied. Thus, we have demonstrated using the CSF that if the human caregiver anticipates the robot's actions, the learning of associations between $|face2\rangle$ and $|act1\rangle$ should be possible. The human anticipation of the robot behavior is a necessary condition for learning since the learning cannot converge in all the other cases. The CSF has added to the CEL, *constraints on the dynamics* of the caregiver behavior. Let us assume that both the CEL and the *constraints on the dynamics* are satisfied. After learning, we have $\Delta t_3 = 0$ in equation 23 and thus the associations learned will verify equ.31:

$$|act1(t)\rangle = c_1(A''|\xi'(|act1(t)\rangle) = c_1(A''|face2\rangle) \tag{31}$$

That means that after the learning convergence and even in an environment without any objects, the robot looks at the same location as the human is looking at. The robot is here able to achieve JA capabilities. Finally the JA will be achieved but this will completely depend on the caregiver ability to maintain the interaction. One more time, the capability to achieve JA in the robot/human/objects system is not within the robot architecture but within the human. The formalization of this kind of temporal constraints appears as necessary to explore the different dynamics which can occur in the proposed experimental setup. We have explicated the *constraints on the dynamics* for learning convergence and also retrieved the experimental results of the JAL architecture but we have also highlighted the limits of the studied architecture due to its lack of internal dynamics. All the *constraints on the dynamics* for learning convergence focus on the human caregiver's behavior: there are no possibility to control the robot reaction time or to obtain the robot to take into account explicitly the other agent's dynamic.

Definition of learning through a temporal window:

We will demonstrate now, that small changes in the robot learning mechanism could influence a lot its learning dynamics: this will enable the learning of associations which could not be learned in the original system . The only changes we want to integrate in the architecture concern the temporal window for learning. Presently, the only associations learned were associations between unconditional and adaptive signal which co-occurred. To enable learning on a temporal window means to enable the learning of associations between unconditional and adaptive signals which does not exactly co-occur but which occur during the same temporal window. Given a temporal window for learning $[t_m, t_M]$, learning converges if at every time t, the *Unconditional* signal is correlated with an *Adaptive* signal occurring during the same temporal window: the correlated *Adaptive* signal must occur at $t + t_0$ with $t_0 \in [t_m, t_M]$. With U an unconditional matrix, A an adaptive matrix for learning, $|s\rangle$ and $|f\rangle$ stimuli depending on time, and c a competition mechanisms, this definition of learning on a temporal window is summarized by equation 32.

$$\forall\ t, \exists\ t_0 \in\ [t_m, t_M]\ /\ \ c(U|s(t)\rangle)\ =\ c(A|f(t+t_0)\rangle) \tag{32}$$

Now, if we consider that the *contingency learning* of the JAL architecture is enabled on a temporal window $[t_m, t_M]$, then, the set of equations 17 characterizing the *constraints on the dynamics* for learning convergence becomes the set 33, with $\Delta t_3 = \Delta t_1 + \Delta t_2$ the duration of a complete interaction loop:

$$\forall\ t, \exists\ t_0 \in\ [t_m, t_M]\ /\ \left\{\begin{array}{lcc} |act1(t)\rangle & = & c_1(A''|\xi'(|act1(t+t_0-\Delta t_3)\rangle, |obj(t+t_0-\Delta t_3)\rangle\rangle)) \\ |act1(t)\rangle & = & c_1(U''_s|obj(t+t_0-\Delta t_1)\rangle) \end{array}\right. \tag{33}$$

Notice that given a temporal window of null width (which corresponds to the original experimental setup of JAL), the set of equations 33 is the same as previously (without using temporal windows, see equ.17). Notice also that if the human caregiver just takes into account the objects, the set of equations 33 does not depend on time and becomes identical to the equation 20. We will consider that the human (*Agent2*) only takes into account the robot's actions ($|act1\rangle$). Then, with a temporal window for learning of width $[t_m, t_M]$, the set of equations which define the *constraints on the dynamics* for learning convergence (equ.23) becomes the set of equations 35:

$$\forall\ t,\ \exists\ t_0\ \in\ [t_m, t_M]\ /\ \begin{cases} |act1(t)\rangle & = & c_1(A''|\xi'(|act1(t+t_0-\Delta t_3)\rangle)) \\ |act1(t)\rangle & = & c_1(U_s''|obj\rangle) \end{cases} \tag{34}$$

According to the *Agent2*'s functionality (ξ' function), this constraint for learning convergence can be written:

$$\forall\ t,\ \exists\ t_0\ \in\ [t_m, t_M]\ /\ |act1(t)\rangle = |act1(t+t_0-\Delta t_3)\rangle \tag{35}$$

Equation 35 shows two things. First, to enable the learning of any association, the temporal window $[t_m, t_M]$ must be wide enough so as to contain Δt_3 (otherwise equ.35 cannot be verified). Second, the temporal window must be as tight as possible to enable the learning of associations to converge. This latter condition is due to the fact that during a temporal window several associations are reinforced (we note that number $n_{[t_m,tM]}$). To allow the learning to converge, one of these associations at least must be more reinforced than the others. We show in *Appendix* 8.3 that to enable the robot to learn associations, $n_{[t_m,t_M]}$ must be inferior to the number of "faces" $|face2\rangle$ the robot may see, n_{face2}. Finally the two following conditions must be satisfied (set of conditions 36) to allow the learning to converge:

$$\begin{cases} \Delta t_3 & \in & [t_m, t_M] \\ n_{[t_m,t_M]} & < & n_{face2} \end{cases} \tag{36}$$

After the learning convergence, we have equation 37:

$$\forall\ t,\ \exists\ t_0\ \in\ [t_m, t_M]\ /\ |act1(t)\rangle = c_1(A''|\xi'(|act1(t+t_0-\Delta t_3)\rangle)) \tag{37}$$

and according to the learning constraints of equ.36, $t_0 = \Delta t_3$ should verify this equation at every time t. Finally we have $|act1(t)\rangle = c_1(A''|\xi'(|act1(t)\rangle))$. That is the same situation as in equation 31: the robot and the human should look at the same location. The robot should achieve JA capability.

7 Conclusion

Theoretical framework like the PDP (Parallel Distributed Processing proposed by [Rumelhart et al., 1989]) and more recent machine learning paradigms (?) are enough for a theoretical study of input/output systems but would not have been sufficient to study a system such as the JAL architecture which interacts with other *Cognitive Systems*. The commonly admitted problem is that when two dynamical systems interact, the dynamics of this interaction is more than the sum of both systems dynamics. The CSF is intended to complete machine learning theories and to provides a way to formalize systems in interaction. Using the CSF, we have theoretically studied the capacity of the JAL (Joint Attention Learning) architecture when interacting with its environment. We have immersed the agent controlled by the JAL architecture in its environment: we have described the sensory inputs of the robot depending on its own actions mediated by the other agent (the human caregiver). We have closed the interaction loop. That has allowed us to formalize the bio-feedback function of the interaction: The robot can learn about its own thanks to its social environment. After the formalization of the interaction loop, we have focused on the determination of the stable states of the interaction with the environment. The research of these stable states leads to define the CEL (Conditions Enabling Learning) and implicates additional *constraints on the dynamics* for the learning convergence: on one hand constraints on the dynamics of the JAL architecture itself and on the other hand constraints on the dynamics of the environment (the human caregiver or another robot). These *constraints on the dynamics* have been shown to be tightly linked to the timing of both the learning mechanism and the interaction loop. We have seen that timing of the different processing is crucial for an agent to be able or not to detect and learn relations between its sensations and its actions. Hence, we have explicated the duration of the interaction loop as a fundamental parameter of human-robot interactions in general. After having fixed the *constraints on the dynamics* enabling the learning convergence, we have generalized these dynamics to the environment not restricted to the CEL and we have thus determine what should be the abilities acquired thanks to the learning, regarding a specific environment, or a specific task to learn. Thus, given a two-agents system and an experimental setup, the CSF characterizes the capability of an agent to learn sensory-motor associations by the media of the other agent. Finally, our theoretical study with the CSF has defined what are the dyad JAL-architecture/environment likely to lead to the achievement of JA: the CSF allowed us to retrieve in a theoretical way the experimental results concerning the JAL architecture [Hosoda et al., 2004].

The characterization of time as the crucial issue for the achievement of Joint Attention by the JAL architecture has allowed us to extend the previous results concerning the JAL architecture and to show what are the necessary and also the best conditions for the convergence of learning. We have shown that a small change on the dynamics of the *Contingency Learning* (enlargement of the temporal window enabling learning) modifies the recursive relation in the *Sensory-Motor* loop and thus the development of the robot. For instance, it allows the robot to learn from being imitated. We have studied the different capabilities of the architecture and among others, what are the benefits of learning in terms of dynamics and behavioral attractors.

Let us take a more general view. Of course the CSF has to be developed, criticized and refine so as to fulfill the ambition of comparing the fundamental principles of various architectures. For instance it must be extended to more complex problems such as long term development including a sequence of emergent stages and more complex dynamics of interaction (turn taking, role switching...). It should also be extended so as to become a measurement tool of the "*Cognitive capabilities*" of an architecture or a model. The solution to represent both the present possible dynamics of an architecture and the possible evolution through time of these possible dynamics has still to be found.

But presently let notice three things. First, this study has given a practical answer to this timing question, proposing a specific enrichment of the JAL architecture, i.e. a change in the learning mechanism timing. Second, the application of the CSF to the study of the JAL architecture has not required any modification of the CSF rules (specific to the JAL architecture): our formal analysis can be re-used for other architectures as it has previously been used for our architectures

[Gaussier et al., 2004, Gaussier et al., 2003, Nadel et al., 2005]. Third, this study has lead to conceptual questions concerning human/robot interactions, and among others, this study has lead to the issue of time scales for both the action and the interaction: what should be the link between the timing of the action and the timing of interaction loop: the interaction loop seems to need much faster frequency than the action decision if we want they feed each other. Considering these three points, this study may be a step toward the ideal goal of a formal tool which could be both conceptual and practical.

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8 Appendix

8.1 The decomposition of a signal

Rule 1: let us consider several signals $|s_i\rangle$ on different channels. Considering a space where these signals are independent, the different channels can be written in a single vector $|s\rangle = (|s_1\rangle, \cdots, |s_n\rangle)$.

$$\begin{pmatrix} |s_1\rangle \\ \vdots \\ |s_n\rangle \end{pmatrix} = |s\rangle$$

Rule 2: let us consider several signals $|s_i\rangle$ processed in several ways (unconditional or not) P_i and giving several outputs $|o_i\rangle$: $\forall i, |o_i\rangle = W_i|s_i\rangle$. Then we can write that each W_i (implying changes without consequence) manages a single signal $|s\rangle$ and gives the $|o_i\rangle$:

$$\forall i, |o_i\rangle = W_i|s\rangle \tag{38}$$

Let us consider several signals $|s_i\rangle$ processed in several ways W_i (unconditional or not) and giving several outputs $|o_i\rangle$: $\forall i, |o_i\rangle = W_i|s_i\rangle$. Considering a first space where the $|s_i\rangle$ are independent and a second where the $|o_i\rangle$ are independent, the equation 38 becomes equation 40:

$$\begin{pmatrix} W_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & W_n \end{pmatrix} \cdot \begin{pmatrix} |s_1\rangle \\ \vdots \\ |s_n\rangle \end{pmatrix} = \begin{pmatrix} |o_1\rangle \\ \vdots \\ |o_n\rangle \end{pmatrix} \tag{39}$$

We can write that each W_i (implying changes without consequence) treats a single signals $|s\rangle$ and provides the $|o_i\rangle$: $\forall i, |o_i\rangle = W_i|s\rangle$. This is given by equation 40

$$\forall\ i,\ W_i' = \begin{pmatrix} 0 & & & & \\ & \ddots & & 0 & \\ & & W_i' & & \\ & 0 & & \ddots & \\ & & & & 0 \end{pmatrix}$$

$$\forall\ i,\ W_i' \cdot |s\rangle = |o_i\rangle \tag{40}$$

Rule 3: let us consider a single signal $|vs\rangle$ processed by several unconditional processes U_i whose outputs are $|s_i\rangle$: $\forall i, U_i|vs\rangle = |s_i\rangle$. We can replace the set of U_i by a single unconditional matrix U and the $|s_i\rangle$ by a single vector $|s\rangle$: $U|vs\rangle = |s\rangle$. Let us consider a single signal $|vs\rangle$ processed by several unconditional processes U_i which outputs are $|s_i\rangle$ (equation 41):

$$\forall i, U_i|vs\rangle = |s_i\rangle \tag{41}$$

Considering a space where the different $|s_i\rangle$ are independent, we can replace the U_i by a single unconditional matrix U and the $|s_i\rangle$ by a single signal $|s\rangle$, and the processing of $|vs\rangle$ is written as in equation 42:

$$\begin{pmatrix} U_1 \\ \vdots \\ U_n \end{pmatrix} \cdot |vs\rangle = \begin{pmatrix} |s_1\rangle \\ \vdots \\ |s_n\rangle \end{pmatrix}$$

$$U \cdot |vs\rangle = |s\rangle \tag{42}$$

Rule 4 (Merging of signals): let us consider several signals $|s_i\rangle$ filtered in several ways (unconditional or not) W_i and giving several outputs $|o_i\rangle$: $\forall i, |o_i\rangle = W_i|s_i\rangle$. The $|s_i\rangle$ can be written as a single signal $|s\rangle$, the filters W_i can be written as a single filter W, and every outputs $|o_i\rangle$ can be written in a single vector $|o\rangle$: $|o\rangle = W|s\rangle$. If W_i includes both conditional and unconditional treatments, the resulting W matrix must be considered as a conditional treatment. In this case, it might be better to separate conditional and unconditional treatment and to use two matrices, U for unconditional and A for adaptive, instead of W. The equation becomes the system of equations 43:

$$\begin{cases} U|s_u\rangle = |o_u\rangle \\ A|s_a\rangle = |o_a\rangle \end{cases} \tag{43}$$

Proof: Let us consider several signals $|s_i\rangle$ processed in several ways (unconditional or not) W_i and giving several outputs $|o_i\rangle$: $\forall i, |o_i\rangle = W_i|s_i\rangle$. It is written as in equation 44:

$$\forall\ i,\ W_i|s_i\rangle = |o_i\rangle \tag{44}$$

Considering a first space where the $|s_i\rangle$ are independent and a second where the $|o_i\rangle$ are independent, the equation 44 becomes equation 45:

$$\begin{pmatrix} W_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & W_n \end{pmatrix} \cdot \begin{pmatrix} |s_1\rangle \\ \vdots \\ |s_n\rangle \end{pmatrix} = \begin{pmatrix} |o_1\rangle \\ \vdots \\ |o_n\rangle \end{pmatrix} \tag{45}$$

We can write the $|s_i\rangle$ as a single signal $|s\rangle$, the filters W_i can be written as one process W on this signal, and every outputs $|o_i\rangle$ can be written in a single vector: $|o\rangle = W|s\rangle$ (see equation 46):

$$W_s = \begin{pmatrix} W_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & W_n \end{pmatrix},\ |s\rangle = \begin{pmatrix} |s_1\rangle \\ \vdots \\ |s_n\rangle \end{pmatrix},\ |o\rangle = \begin{pmatrix} |o_1\rangle \\ \vdots \\ |o_n\rangle \end{pmatrix}$$

$$W_s \cdot |s\rangle = |o\rangle \tag{46}$$

Rule 5: let us consider a single signal $|s\rangle$ filtered by an unconditional matrix U (linear application) which extracts several signals $|s_i\rangle$. We then can replace $|s\rangle$ by the several signals $|s_i\rangle$ extracted.

Let us consider a single signal $|s\rangle$ filtered by an unconditional matrix U (linear application) which extracts several signals $|s_i\rangle$ (see equ.47):

$$U\ |s\rangle = \begin{pmatrix} |s_1\rangle \\ \vdots \\ |s_n\rangle \end{pmatrix} \tag{47}$$

U can thus be written as the sum of n matrix U_i (see equ.48:

$$U\ =\ \begin{pmatrix} U_1 \\ \vdots \\ U_n \end{pmatrix} = \begin{pmatrix} |U_1\rangle \\ \vdots \\ 0 \end{pmatrix} + \cdots + \begin{pmatrix} 0 \\ \vdots \\ |U_n\rangle \end{pmatrix} \tag{48}$$

The flow of information represented by $|s\rangle$ can thus be decomposed in n signals implying a linear transformation (see 49):

$$\begin{aligned} U|s\rangle\ &=\ \begin{pmatrix} U_1 \\ \vdots \\ U_n \end{pmatrix}|s\rangle\ =\ \begin{pmatrix} |U_1\rangle \\ \vdots \\ 0 \end{pmatrix}|s\rangle + \cdots + \begin{pmatrix} 0 \\ \vdots \\ |U_n\rangle \end{pmatrix}|s\rangle \\ &=\ \begin{pmatrix} |s_1\rangle \\ \vdots \\ 0 \end{pmatrix} + \cdots + \begin{pmatrix} 0 \\ \vdots \\ |s_n\rangle \end{pmatrix} \end{aligned} \tag{49}$$

Finally the signal $|s\rangle$ can be written as n independent signals $|s_i\rangle$ without any change in the meaning of the computation.

8.2 Simplification of compound functions

Simplification of compound functions: Considering that the input of a function ξ can be written as the product of an *unconditional matrix* U by a vector $|v\rangle$ then $\xi(U|v\rangle) = \xi'(|v\rangle)$ with ξ' the compound function $\xi \circ U$.

Considering that if M_1 and M_2 are two matrices (unconditional or not) and c is a competition mechanism, then a matrix M does exist which verifies $c|M = c|M_1|c|M_2$ (M unconditional if both M_1 and M_2 are, adaptive in every other cases).

8.3 Convergence of learning given a temporal window

The *Agent1*'s architecture does not present any property of regularity between its successive actions: given an action $|act1_0\rangle$ every preceding and following actions have the same chance to occur. Let consider a specific association between a face pattern of the human $|face2_0\rangle$ and an action of the robot $|act1_0\rangle$. When $|act1_0\rangle$ occurs at time t_0, according to the learning window, the reinforced associations are those which link $|act1_0\rangle$ with the set of human gaze directions $\{|face2(t_0+t)\rangle,\ t\ \in\ [t_m, t_M]\}$. We write $n_{[t_m,t_M]}$ the number of element of this latter set. Given an action $|act1_0\rangle$, $n_{[t_m,t_M]}$ associations with this action are reinforced. Now, we name n_{face2} the number of possible vectors $|face2\rangle$ which may be presented as inputs of the architecture (n_{face2} is the number of different gaze directions the human may demonstrate). If every of these inputs have the same chance to occur, then each one of them has $\frac{n_{[t_m,t_M]}}{n_{face2}}$ chances to be in $\{|face2(t_0+t)\rangle,\ t\ \in\ [t_m, t_M]\}$ (for a specific occurrence of $|act1_0\rangle$). If $|face2_0\rangle \in \{|face2(t_0+t)\rangle,\ t\ \in\ [t_m, t_M]\}$ then each one of the $n_{[t_m,t_M]} - 1$ possible irrelevant actions has $\frac{n_{[t_m,t_M]}-1}{n_{face2}-1}$ chances to be in the set and thus to be reinforced. A mean reinforcement $\overline{R}$ for each occurrence of $|act1_0\rangle$ can thus be defined for each association: $\overline{R}_{good}$ for relevant ones and $\overline{R}_{bad}$ for irrelevant ones. If $|face2_0\rangle \in \{|face2(t_0+t)\rangle,\ t\ \in\ [t_m, t_M]\}$, then $\overline{R}_{good} \geq 1$ and $\overline{R}_{bad} \leq \frac{n_{[t_m,t_M]}-1}{n_{face2}-1}$. If $|face2_0\rangle \notin \{|face2(t_0+t)\rangle,\ t\ \in\ [t_m, t_M]\}$, $\overline{R} = \overline{R}_{good} = \overline{R}_{bad} = \frac{n_{[t_m,t_M]}}{n_{face2}}$. The condition for learning convergence is that $\overline{R}_{good} > \overline{R}_{bad}$. A condition for the learning convergence can thus be defined by the inequality 50:

$$\overline{R}_{bad} < \overline{R}_{good} \quad \Rightarrow \quad \frac{n_{[t_m,t_M]}-1}{n_{face2}-1} < 1 \quad \Rightarrow \quad n_{[t_m,t_M]} - 1 < n_{face2} - 1 \quad \Rightarrow \quad n_{[t_m,t_M]} < n_{face2} \tag{50}$$

Finally to enable the robot to learn the association between its own action $|act1_0\rangle$ and the human caregiver gaze direction $|face2_0\rangle$, the two following conditions must be satisfied (set of conditions 51):

$$\begin{cases} |face2_0\rangle \in \{|face2(t_0+t)\rangle,\ t\ \in\ [t_m, t_M]\} \\ n_{[t_m,t_M]}\ <\ n_{face2} \end{cases} \tag{51}$$